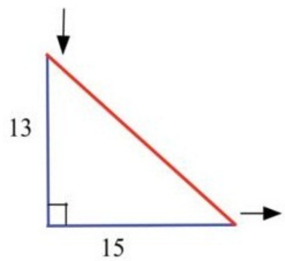
Name: ________________

PROBLEMS

1. An expandable sphere is being filled with liquid at a constant rate from a tap (imagine a water balloon connected to a faucet). When the radius of the sphere is 3 inches, the radius is increasing at 2 inches per minute. How fast is the liquid coming out of the tap? ($V = \frac{4}{3}\pi r^3$)



3. One hour later the right triangle in Problem 2 is 15 inches long and 13 inches high (Fig. 9), and the base and height are changing at the same rate as in Problem 2.
   (a) Is the area increasing or decreasing now?
   (b) Is the hypotenuse increasing or decreasing now?
   (c) Is the perimeter increasing or decreasing now?

5. The length of a 12 foot by 8 foot rectangle is increasing at a rate of 3 feet per second and the width is decreasing at 2 feet per second (Fig. 11).
   (a) How fast is the perimeter changing?
   (b) How fast is the area changing?

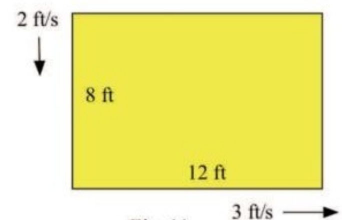


Fig. 11

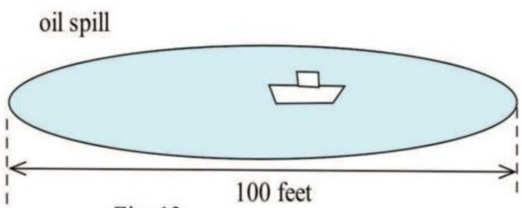


Fig. 13

7. An oil tanker in Puget Sound has sprung a leak, and a circular oil slick is forming (Fig. 13). The oil slick is 4 inches thick everywhere, is 100 feet in diameter, and the diameter is increasing at 12 feet per hour. Your job, as the Coast Guard commander or the tanker's captain, is to determine how fast the oil is leaking from the tanker.

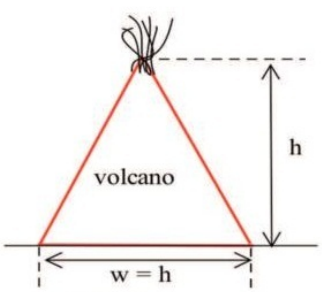


Fig. 15

9. Lava flowing from a hole at the top of a hill is forming a conical mountain whose height is always the same as the width of its base (Fig. 15). If the mountain is increasing in height at 2 feet per hour when it is 500 feet high, how fast is the lava flowing (how fast is the volume of the mountain increasing)? ($V = \frac{1}{3}\pi r^2 h$)

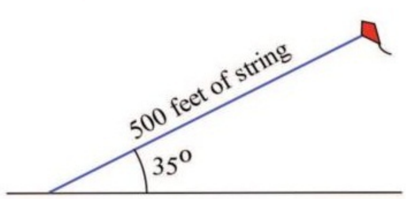


Fig. 18

13. The string of a kite is perfectly taut and always makes an angle of 35° above horizontal (Fig. 18).
   (a) If the kite flyer has let out 500 feet of string, how high is the kite?
   (b) If the string is let out at a rate of 10 feet per second, how fast is the kite's height increasing?

15. The 8 foot diameter of a spherical gas bubble is increasing at 2 feet per hour, and the 12 foot long edges of a cube containing the bubble are increasing at 3 feet per hour. Is the volume contained between the spherical bubble and the cube increasing or decreasing? At what rate?

17. The snow in a hemispherical pile melts at a rate proportional to its exposed surface area (the surface area of the hemisphere). Show that the height of the snow pile is decreasing at a constant rate.

19. Define A(x) to be the **area** bounded by the x and y axes, the horizontal line $y = 5$, and a vertical line at x (Fig. 20).

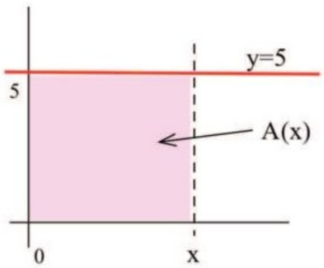


Fig. 20

(a) Find a formula for A as a function of x.

(b) Determine $\frac{d\,A(x)}{dx}$ when $x = 1, 2, 4$ and 9.

(c) Suppose x is a function of time, $x(t) = t^2$, and find a formula for A as a function of t.

(d) Determine $\frac{d\,A}{dt}$ when $t = 1, 2$, and 3.

(e) Suppose $x(t) = 2 + \sin(t)$. Find a formula for A(t) and determine $\frac{d\,A}{dt}$.

21. You are walking along a sidewalk toward a 40 foot wide sign which is adjacent to the sidewalk and perpendicular to it (Fig. 22).

(a) If your viewing angle θ is 10°, then how far are you from the nearest corner of the sign?

(b) If your viewing angle is 10° and you are walking at 25 feet per minute, then how fast is your viewing angle changing?

(c) If your viewing angle is 10° and is increasing at 2° per minute, then how fast are you walking?

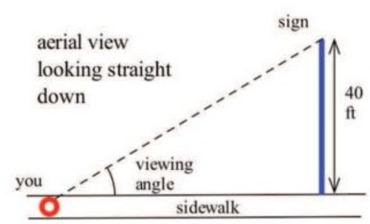


Fig. 22