Pre-Calculus 11 Final Exam Review

Name: _____

1. Find the general term of the arithmetic sequence.

   a. $3, 7, 11, 15, \ldots$ b. $37, 29, 21, 13, \ldots$ c. $3, 5\frac{1}{4}, 7\frac{1}{2}, 9\frac{3}{4}, \ldots$

2. Find the general term and the indicated term of the arithmetic sequence with these terms:

   a. $t_6 = 41$ and $t_{11} = 71$, t_{20} b. $t_4 = -3$ and $t_9 = -18$, t_{17}

3. For the sequence $-5, -1, 3, 7, \ldots$ which term is 147?

4. Determine the sum of the first 25 terms of the arithmetic series.

   a. $3 + 6 + 9 + 12 + \cdots$ b. $5 + 1 + (-3) + (-7) + \cdots$

5. Determine the sum of the series.

   a. $5 + 8 + 11 + \cdots + 47$ b. $(-37) + (-31) + (-25) + \cdots + 95$

6. A triangular display of soup cans has 45 cans in the 9th row and 27 cans in the 15th row. If the number of cans in each row is the term of an arithmetic sequence, how many cans are in the display?

7. In a geometric sequence, $t_2 = 15$ and $t_7 = 3645$. Find t_5.

8. Fill in the blanks for the following geometric sequences.

   a. -2, ___, ___, 54, ___ b. -1, ___, ___, ___, -625

9. A population of bacteria doubles every 20 minutes. If a single bacterium started producing at midnight, how many would there be at noon?

10. Determine if each geometric series converges or diverges.

    a. $a_1 = -3, r = 4$ b. $81 + 27 + 9 + 3 + \cdots$

11. Evaluate each infinite series.

    a. $a_1 = 1, r = \frac{1}{2}$ b. $1 + 0.5 + 0.25 + 0.125 + \cdots$

12. Determine the common ratio of the infinite geometric series with $a_1 = -4$ and $S = -\frac{16}{5}$.

13. Evaluate each geometric series described:

    a. $1 + 2 + 4 + 8 + \cdots; n = 6$ b. $1 - 4 + 16 - 64 + \cdots; n = 9$

14. Determine the number of terms n in each geometric series.

    a. $a_1 = -2, r = 5, S_n = -62$ b. $-4 + 16 - 64 + 256 + \cdots; S_n = 52428$

15. Find the exact value of each trigonometric function.

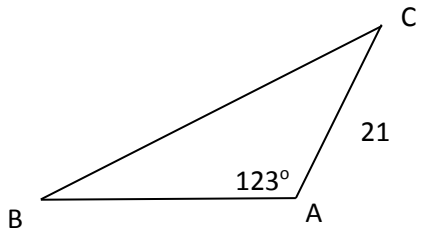
a. $\sin 0°$ b. $\tan 240°$ c. $\cos 225°$ d.$\sin 330°$

16. Given that $0° \leq \theta < 360°$, solve for angle θ. (Exact values)

a. $\sin\theta = -\frac{\sqrt{3}}{2}$ b. $\cos\theta = \frac{1}{2}$ c. $\sin\theta = \frac{1}{\sqrt{2}}$ d. $\tan\theta = -1$

17. Solve each Triangle. Round your solutions to the nearest whole.

a.



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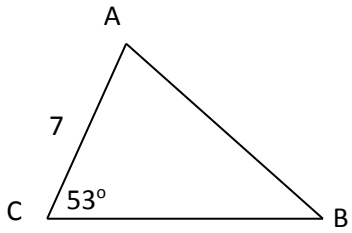

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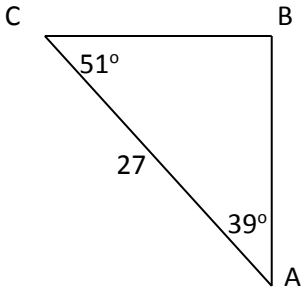

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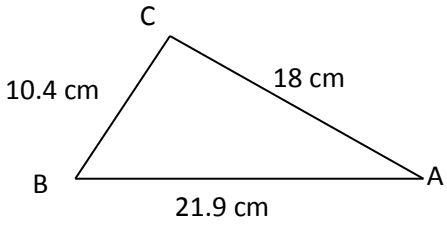


18. a. Graph the following quadratic functions manually. (Clearly indicate at least five points.)

i.) $y = -\frac{1}{2}(x-3)^2 + 4$ ii.) $y = 2(x+5)^2 - 6$ iii.) $y = x^2 - 6x + 1$

b. For each of the functions above, determine the:

Vertex

Equation of the axis of symmetry

Domain & Range

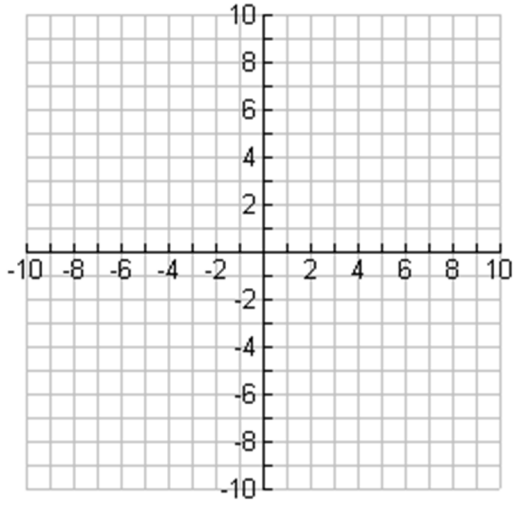
Number of zeros

Maximum or minimum value

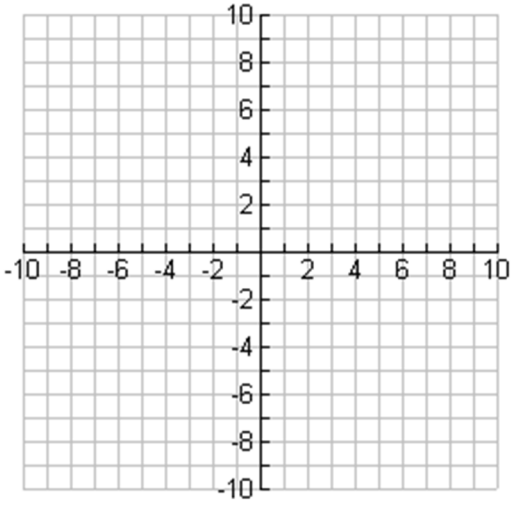
Direction of opening

19. Sketch each parabola.

a. $f(x) = -(x-6)^2 + 2$



b. $f(x) = -(x-5)^2 - 1$



20. Change each quadratic function to vertex form.

a. $y = x^2 - 2x - 5$

b. $y = -x^2 - 14x - 59$

c. $y = 2x^2 + 36x + 170$

d. $-(y+1) = (x-4)^2$

21 Write the equations of the quadratic functions with the following features:

a. Has zeros 4, and -2, is congruent to $y = x^2$ and opens up.

b. Has vertex (5, 4), and passes through the point (6, 2)

c. Has vertex (-4, 7), is congruent to $y = -3x^2 + 4$, and opens down.

22. The height of a cannonball, *h*, can be described as a function of its horizontal distance from the cannon, *d*, by the equation: $h = -\frac{3}{625}d^2 + 2.4d$.

a. What is the maximum height of the cannonball?

b. How far does it travel before it hits the ground?

c. How high is it 20 m from the cannon?

d. If there is a 60m Tree in the line of fire 400m from the cannon, will the cannon hit the tree?

e. Over what distance is the height of the cannon greater than 100m?

23. Solve the following algebraically:

a. $x^2 + 6x - 16 = 0$ b. $4x^2 - 12x - 40 = 0$ c. $7n + 4 = 2n^2$ d. $9x^2 - 24x + 16 = 0$
e. $8p^2 - 50 = 0$ f. $2x^2 + 3x = 3 - 2x$ g. $2y^2 = 7y + 30$ h. $2n^2 + 10 = 108$
i. $(m-3)^2 = 6(m+5)$ j. $x^2 = -4x$ k $(x-3)^2 - 49 = 0$

24. Without solving determine the nature of the roots for each of the following quadratic equations.

a. $3x^2 - 5x + 12 = 0$ b. $3x^2 - 6x + 2 = 0$

25. For what values of *k* does $2x^2 - 6x + k = 0$ have no real roots?

26. Find *k* so that $2x^2 + x + 2k + 1 = 0$ has no real roots.

27. Simplify each radical expression. Rationalize when needed.

a. $-3\sqrt{12} + 3\sqrt{3} + 3\sqrt{20}$ b. $2\sqrt[3]{6} - \sqrt[6]{6} + 3\sqrt[3]{6} - 3\sqrt[3]{384}$ c. $4\sqrt{15}(\sqrt{6} + \sqrt{5})$

d. $\left(\sqrt{3} + \sqrt{5x}\right)(\sqrt{3} - 5\sqrt{5x})$ e. $\frac{3}{\sqrt{5x}-3}$ f. $\frac{\sqrt[3]{10}}{\sqrt[3]{32}}$

g. $\frac{4\sqrt{2}}{3\sqrt{5}}$ h. $\sqrt[6]{448x^7y^7}$

28. Solve the following equations algebraically. Indicate the restrictions and any extraneous roots.

a. $\sqrt{x-2} = 4 - x$ b. $\sqrt{3x-5} = -2$ c. $\sqrt{2x+5} - \sqrt{x+3} = 0$

29. Determine the non-permissible values and simplify the rational expression.

a. $\frac{u-v}{8v} + \frac{6u-3v}{8v}$ b. $\frac{x+2}{2x^2+13x+20} - \frac{x+3}{2x^2+13x+20}$ c. $\frac{3}{b-8} + \frac{7}{b+3}$

d. $\frac{x^2+x-6}{x^2+8x+15}$ e. $\frac{x-8}{(x+6)(x-8)} \cdot \frac{4x(x+10)}{x+10}$ f. $\frac{2b^2-12b}{b+5} \div \frac{b-6}{b+5}$

30. Solve each equation algebraically. State the non-permissible values.

a. $\frac{x+1}{x-2} = \frac{2x+3}{x-3}$ b. $\frac{3}{x} - \frac{4}{x-3} = 2$ c. $\frac{x}{x-2} + \frac{1}{x+2} = \frac{8}{x^2-4}$

d. $\frac{5}{2y} - \frac{7}{10y} = \frac{3}{y-2}$ e. $\frac{1}{r-2} + \frac{1}{r^2-7r+10} = \frac{6}{r-2}$ f. $\frac{5}{n^3+5n^2} = \frac{4}{n+5} + \frac{1}{n^2}$

31. Solve each equation.

a. $|p-3| = 3$ b. $|6+5p| = 14$ c. $|-5x|+4 = -11$

d. $-8|3-8k| = 40$ e. $|x+6|+3 = 5x-1$

32. Plot the following absolute value function and express each as a piecewise function.

a. $f(x) = |2x-5|$ b. $f(x) = |-x+3|$ c. $f(x) = |(x-5)^2+3|$ d. $f(x) = |-(x-1)^2+3|$

33. For each reciprocal function, state the equation of the asymptote, the coordinates of the invariant points and sketch the function.

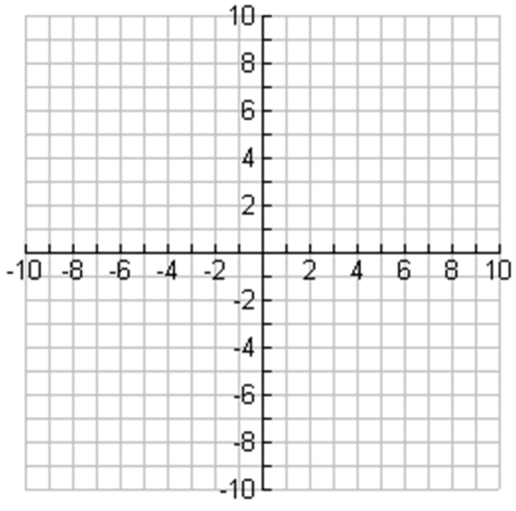
a. $f(x) = \frac{2}{2x-3}$ b. $f(x) = \frac{1}{2x^2-11x+12}$

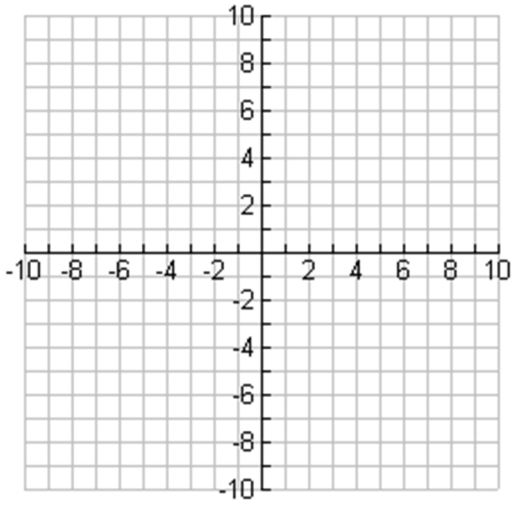
34. Solve each system.

a. $\begin{cases} y = 3x^2-2x-3 \\ y = -2x^2-15x+2 \end{cases}$ b. $\begin{cases} y = -x^2-2x+2 \\ 4x^2-6x+2y = -8 \end{cases}$ c. $\begin{cases} y = 2x-2 \\ -4 = y+7x^2-7x \end{cases}$

35. Sketch the solution of each inequality

a. $y < x^2+2x-2$ b. $y \leq -x^2+5x-2$





Solutions:

1. a. $t_n = 4n - 1$ b. $t_n = -8n + 45$ c. $t_n = \frac{9}{4}n + \frac{3}{4}$ 2. a. $t_n = 6n + 5, t_{20} = 125$

b. $t_n = -3n + 9, t_{17} = -42$ 3. 39th 4.a. 975 b. -1075 5. a. 390 b. 667

6. 828 7. 405 8. a. 6, -18, -162 b.5, -25, 125 9. 2^{36} 10.a. diverges b. converges

11. a. 2 b. 2 12. $-\frac{1}{4}$ 13. a. 63 b. 5242914. a. 3 b. 8 15. a. 0 b. $\sqrt{3}$

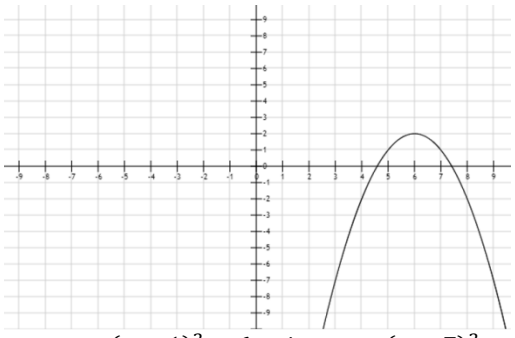
c. $-\frac{1}{\sqrt{2}}$ d. $-\frac{1}{2}$ 16. a. 240300 b. 60300 c. 45124 d. 135315 17. a. $a = 45, B = 23, C = 34$

b. $a = 10, A = 83, c = 8$ c. $a = 17, c = 21, B = 90$ d. $C = 97, A = 28, B = 55$

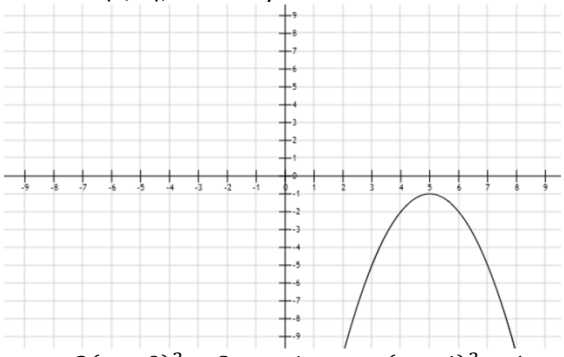
18 a. i) (3, 4), $x = 3, x \in R, y \leq 4$, 2 zeroes, max = 4, down b. ii) (-5, -6), $x = -5, x \in R, y \geq -6$, 2 zeroes, min = -6, up

c. iii) (3, -8), $x = 3, x \in R, y \geq -8$, 2 zeroes, min = -8, up

19a. Vertex (6, 2), Axis of Sym: x = 6



b. Vertex (5,-1), Axis of Sym: x = 5



20. a.$y = (x - 1)^2 - 6$ b. $y = -(x + 7)^2 - 10$ c. $y = 2(x + 9)^2 + 8$ d. $y = -(x - 4)^2 - 1$

21. a. $y = (x - 1)^2 - 9$ b. $y = -2(x - 5)^2 + 4$ c. $y = -3(x + 4)^2 + 7$ 22. a. 300 m b. 500m