

## Chapter 6 BLM Answers

### BLM 6-1 Chapter 6 Prerequisite Skills

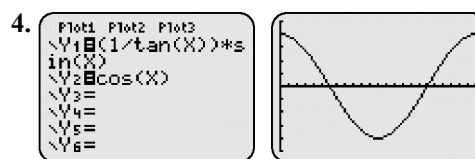
- II
    - IV
    - III
  - $45^\circ$
    - $\frac{2\pi}{7}$
    - $\frac{\pi}{4}$
- $310^\circ$
  - $-\frac{\pi}{2}, \frac{7\pi}{2}$
- $488.4^\circ, -231.6^\circ, -591.6^\circ$
  - $-\frac{7\pi}{4}, \frac{9\pi}{4}$
- $115^\circ + 360^\circ n; n \in \mathbb{I}$
  - $\frac{7\pi}{3} + 2\pi n; n \in \mathbb{I}$
  - $13.4 \text{ rad} + 2\pi n \text{ rad}; n \in \mathbb{I}$
- $252^\circ$
  - $\frac{11\pi}{6}$
  - $269.3^\circ$
  - $-0.88$
- negative
  - negative
  - positive
  - positive
  - negative
  - negative
- $\frac{1}{2}$
  - $\frac{2\sqrt{3}}{3}$
  - $\frac{-\sqrt{3}}{3}$
  - $\frac{3}{4}$
- 0.58
  - 1.06
  - 0.41
  - 225.68
- $\frac{5\pi}{6}$
  - $\frac{8\pi}{3}$
  - $\frac{5\pi^2}{8}$
  - $\frac{-\pi}{24}$
- $x(x-1)$
  - $(x-4)(x+1)$
  - $(x-y)(x+y)$
  - $3(x+1)(x-2)$
  - $2x(x-2)(x+5)$
- $\theta \neq \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$
  - $\theta \neq -\frac{\pi}{2}, -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$
- $\theta \neq$  odd integral multiples of  $\frac{\pi}{2}$
  - $\theta \neq$  odd integral multiples of  $\frac{\pi}{2}$ , odd integral multiples of  $\frac{\pi}{4}$
- $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$
- $\theta = \frac{\pi}{6} + 2\pi n; n \in \mathbb{I}$  or  $\theta = \frac{11\pi}{6} + 2\pi n; n \in \mathbb{I}$

### BLM 6-2 Section 6.1 Extra Practice

- $x \neq \frac{\pi}{2} + \pi n; n \in \mathbb{I}$
  - $x \neq \pi n; n \in \mathbb{I}$  and  $x \neq \frac{\pi}{2} + \pi n; n \in \mathbb{I}$
  - $x \neq 2\pi n; n \in \mathbb{I}$  and  $x \neq \frac{\pi}{2} + \pi n; n \in \mathbb{I}$
  - $x \neq \frac{\pi}{2} n; n \in \mathbb{I}$

2.  $\theta \neq \pi n; n \in \mathbb{I}$

3. a)  $\cos x$  b)  $\cos x$  c)  $\sin x$  d)  $\tan x$



5. a) 2 b) 1 c)  $\sec^2 x$  d) 1 e)  $\sin x \cos x$  f) 1

6. a) may be an identity  
b) not an identity c) not an identity

7. a)  $\cot x$  b)  $\sec x$  c)  $\csc x$

8. Left side =  $\sin^4\left(\frac{\pi}{6}\right) - \cos^4\left(\frac{\pi}{6}\right)$

$$= \frac{1}{16} - \frac{9}{16}$$

$$= -\frac{1}{2}$$

Right side =  $2 \sin^2\left(\frac{\pi}{6}\right) - 1$

$$= -\frac{1}{2}$$

= Left side



$$\begin{aligned}
 9. \text{ Left side} &= \sec\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) \\
 &= \frac{1}{\cos\left(\frac{\pi}{4}\right)} + \frac{\cos\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} \\
 &= \frac{2}{\sqrt{2}} + 1
 \end{aligned}$$

$$\begin{aligned}
 \text{Right side} &= 1 + \sec\left(\frac{\pi}{4}\right) \\
 &= 1 + \frac{1}{\cos\left(\frac{\pi}{4}\right)} \\
 &= 1 + \frac{2}{\sqrt{2}} \\
 &= \text{Left side}
 \end{aligned}$$

10. a)

$$\begin{aligned}
 \text{LS} &= \cos\left(\frac{\pi}{6}\right) \div \left(\sec\left(\frac{\pi}{6}\right) - 1\right) + \cos\left(\frac{\pi}{6}\right) \div \left(\sec\left(\frac{\pi}{6}\right) + 1\right) \\
 &= \frac{\sqrt{3}}{2} \div \left(\frac{2}{\sqrt{3}} - 1\right) + \frac{\sqrt{3}}{2} \div \left(\frac{2}{\sqrt{3}} + 1\right) \\
 &= \frac{\sqrt{3}}{2} \div \left(\frac{2 - \sqrt{3}}{\sqrt{3}}\right) + \frac{\sqrt{3}}{2} \div \left(\frac{2 + \sqrt{3}}{\sqrt{3}}\right) \\
 &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2 - \sqrt{3}} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2 + \sqrt{3}} \\
 &= \frac{3}{4 - 2\sqrt{3}} + \frac{3}{4 + 2\sqrt{3}} \\
 &= \frac{12 + 6\sqrt{3} + 12 - 6\sqrt{3}}{16 - 12} \\
 &= 6
 \end{aligned}$$

$$\begin{aligned}
 \text{RS} &= 2\cot^2\left(\frac{\pi}{6}\right) \\
 &= \frac{2}{\tan^2\left(\frac{\pi}{6}\right)} \\
 &= 2 \div \frac{1}{3} \\
 &= 6 \\
 &= \text{LS}
 \end{aligned}$$

b)  $x \neq 0^\circ, 180^\circ$

$$11. \cos^2 x + \sin^2 x = 1$$

$$\begin{aligned}
 \frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} &= \frac{1}{\sin^2 x} \\
 \cot^2 x + 1 &= \csc^2 x
 \end{aligned}$$

**BLM 6-3 Section 6.2 Extra Practice**

1. a)  $\sin 63^\circ$  b)  $\cos 17^\circ$

c)  $\cos\left(-\frac{\pi}{6}\right)$  d)  $\sin\left(\frac{\pi}{12}\right)$

2. a)  $\frac{1}{2}$  b)  $\frac{\sqrt{3}}{2}$  c) 1 d)  $\frac{1}{\sqrt{2}}$

3. a)  $\sin \frac{\pi}{3}$  b)  $\cos \frac{2\pi}{3}$

c)  $\cos 30^\circ$  d)  $\tan \frac{\pi}{3}$

4. a)  $\cos A$  b)  $-\sin A$  c)  $-\sin A$  d)  $\cos A$

5. a)  $\cos A$  b)  $-\cos A$  c)  $\cos A$  d)  $-\sin A$

6. a)  $\cos \theta$  b)  $\cos(4x)$  c)  $-\sin \theta$  d)  $-\sin \theta$

7. a)  $-\frac{1}{2}$  b)  $2 - \sqrt{3}$  c)  $\frac{\sqrt{3} + 1}{2\sqrt{2}}$  d)  $-\frac{\sqrt{3}}{2}$

8. a) true b) false c) true d) false

9. a)  $\frac{56}{65}$  b)  $\frac{63}{65}$  c)  $\frac{-119}{169}$  d)  $\frac{24}{25}$

10.  $-\frac{120}{169}$

**BLM 6-4 Section 6.3 Extra Practice**

1. a)  $\frac{1}{\sin x}$  b)  $\frac{1}{\sin^2 x}$  c)  $\cos x$

2. a)  $\tan x$  b)  $\frac{\sin x - 2}{5}$  c)  $\frac{\cos x + 2}{7}$

d)  $\sin x - 1$

3. a) Example:

$$\text{Left side} = \csc^2 x (1 - \cos^2 x)$$

$$= \frac{1}{\sin^2 x} (\sin^2 x)$$

$$= 1$$

$$= \text{Right side}$$



**b) Example:**

$$\begin{aligned}\text{Left side} &= (\tan x - 1)^2 \\ &= \tan^2 x - 2 \tan x + 1 \\ &= \frac{\sin^2 x - 2 \sin x \cos x + \cos^2 x}{\cos^2 x}\end{aligned}$$

$$\begin{aligned}\text{Right side} &= \frac{1}{\cos^2 x} - \frac{2 \sin x}{\cos x} \\ &= \frac{1 - 2 \sin x \cos x}{\cos^2 x} \\ &= \frac{\sin^2 x - 2 \sin x \cos x + \cos^2 x}{\cos^2 x} \\ &= \text{Left side}\end{aligned}$$

**c) Example:**

$$\text{Right side} = \cos x$$

$$\begin{aligned}\text{Left side} &= \frac{\sin^2 x + \cos^2 x}{\sec x} \\ &= 1 \div \frac{1}{\cos x} \\ &= \cos x \\ &= \text{Right side}\end{aligned}$$

**4. a) Example:**

$$\text{Right side} = \tan x$$

$$\begin{aligned}\text{Left side} &= \frac{1 + \tan x}{1 + \cot x} \\ &= 1 + \frac{\sin x}{\cos x} \div \left( 1 + \frac{\cos x}{\sin x} \right) \\ &= \frac{\cos x + \sin x}{\cos x} \times \frac{\sin x}{\sin x + \cos x} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \\ &= \text{Right side}\end{aligned}$$

**b) Example:**

$$\text{Right side} = \cot x$$

$$\begin{aligned}\text{Left side} &= \frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} \\ &= \frac{1}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x} \\ &= \frac{\cos^2 x}{\sin x \cos x} \\ &= \cot x \\ &= \text{Right side}\end{aligned}$$

**c) Example:**

$$\text{Right side} = \csc x$$

$$\begin{aligned}&= \frac{1}{\sin x} \\ \text{Left side} &= \frac{\cot x + \tan x}{\sec x} \\ &= \cos x \left( \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} \right) \\ &= \frac{\cos^2 x + \sin^2 x}{\sin x} \\ &= \frac{1}{\sin x} \\ &= \text{Right side}\end{aligned}$$

**5. a) Example:**

$$\begin{aligned}\text{Left side} &= \frac{\csc x + \cot x}{\tan x + \sin x} \\ &= \frac{1 + \cos x}{\sin x} \div \frac{\sin x + \sin x \cos x}{\cos x} \\ &= \frac{1 + \cos x}{\sin x} \times \frac{\cos x}{\sin x(1 + \cos x)} \\ &= \frac{\cos x}{\sin^2 x}\end{aligned}$$

$$\text{Right side} = \cot x \csc x$$

$$\begin{aligned}&= \frac{\cos x}{\sin^2 x} \\ &= \text{Left side}\end{aligned}$$

**b) Example:**

$$\text{Right side} = \tan x$$

$$\begin{aligned}\text{Left side} &= \frac{\sin x + \tan x}{\cos x + 1} \\ &= \frac{\sin x \cos x + \sin x}{\cos x} \times \frac{1}{\cos x + 1} \\ &= \frac{\sin x(\cos x + 1)}{\cos x} \times \frac{1}{\cos x + 1} \\ &= \tan x \\ &= \text{Right side}\end{aligned}$$

**c) Example:**

$$\text{Right side} = \cot x$$

$$\begin{aligned}\text{Left side} &= \frac{\cos x + 1}{\sin x + \tan x} \\ &= (\cos x + 1) \times \frac{\cos x}{\sin x(\cos x + 1)} \\ &= \cot x \\ &= \text{Right side}\end{aligned}$$



**6. a) Example:**

$$\text{Right side} = \frac{1 + \sin x}{\cos x}$$

$$\begin{aligned} \text{Left side} &= \frac{\cos x}{1 - \sin x} \\ &= \frac{\cos x(1 + \sin x)}{(1 - \sin x)(1 + \sin x)} \\ &= \frac{\cos x(1 + \sin x)}{1 - \sin^2 x} \\ &= \frac{\cos x(1 + \sin x)}{\cos^2 x} \\ &= \frac{1 + \sin x}{\cos x} \\ &= \text{Right side} \end{aligned}$$

**b) Example:**

$$\text{Left side} = \frac{1 + \cos x}{\sin x}$$

$$\begin{aligned} \text{Right side} &= \frac{\sin x}{1 - \cos x} \times \frac{1 + \cos x}{1 + \cos x} \\ &= \frac{\sin x(1 + \cos x)}{\sin^2 x} \\ &= \frac{1 + \cos x}{\sin x} \\ &= \text{Left side} \end{aligned}$$

**c) Example:**

$$\text{Right side} = 2 \cot^2 x$$

$$= \frac{2 \cos^2 x}{\sin^2 x}$$

$$\begin{aligned} \text{Left side} &= \frac{\cos x}{\sec x - 1} + \frac{\cos x}{\sec x + 1} \\ &= \cos x \div \frac{1 - \cos x}{\cos x} + \cos x \div \frac{1 + \cos x}{\cos x} \\ &= \frac{\cos^2 x}{1 - \cos x} + \frac{\cos^2 x}{1 + \cos x} \\ &= \frac{2 \cos^2 x}{1 - \cos^2 x} \\ &= \frac{2 \cos^2 x}{\sin^2 x} \\ &= \text{Right side} \end{aligned}$$

**7. a) Example:**

$$\text{Right side} = \cos^2 x - \sin^2 y$$

$$\begin{aligned} \text{Left side} &= \cos(x + y)\cos(x - y) \\ &= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y) \\ &= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y \\ &= \cos^2 x(1 - \sin^2 y) - \sin^2 y(1 - \cos^2 x) \\ &= \cos^2 x - \sin^2 y \cos^2 x - \sin^2 y + \sin^2 y \cos^2 x \\ &= \cos^2 x - \sin^2 y \\ &= \text{Right side} \end{aligned}$$

**b) Example:**

$$\text{Right side} = \cot x$$

$$= \frac{\cos x}{\sin x}$$

$$\begin{aligned} \text{Left side} &= \frac{1 + \cos 2x}{\sin 2x} \\ &= \frac{1 + 2 \cos^2 x - 1}{2 \sin x \cos x} \\ &= \frac{\cos x}{\sin x} \\ &= \text{Right side} \end{aligned}$$

**c) Example:**

$$\text{Right side} = (\sin x + \cos x)^2$$

$$\begin{aligned} &= \sin^2 x + 2 \sin x \cos x + \cos^2 x \\ &= 1 + 2 \sin x \cos x \end{aligned}$$

$$\begin{aligned} \text{Left side} &= 1 + \sin 2x \\ &= 1 + 2 \sin x \cos x \\ &= \text{Right side} \end{aligned}$$

**d) Example:**

$$\text{Right side} = \frac{2}{1 + \cos 2x}$$

$$= \frac{2}{1 + 2 \cos^2 x - 1}$$

$$= \frac{1}{\cos^2 x}$$

$$\text{Left side} = \sec^2 x$$

$$= \frac{1}{\cos^2 x}$$

$$= \text{Right side}$$



**8. a)** Verify for  $x = 30^\circ$ :

$$\begin{aligned} \text{Left side} &= \sec^4 30^\circ - \sec^2 30^\circ \\ &= \frac{16}{9} - \frac{4}{3} \end{aligned}$$

$$= \frac{4}{9}$$

$$\text{Right side} = \tan^4 30^\circ + \tan^2 30^\circ$$

$$= \frac{1}{9} + \frac{1}{3}$$

$$= \frac{4}{9}$$

$$= \text{Left side}$$

Example:

$$\begin{aligned} \text{Left side} &= \sec^4 x - \sec^2 x \\ &= \frac{1}{\cos^2 x} \left( \frac{1 - \cos^2 x}{\cos^2 x} \right) \end{aligned}$$

$$= \frac{\sin^2 x}{\cos^4 x}$$

$$\text{Right side} = \tan^4 x + \tan^2 x$$

$$= \tan^2 x (\tan^2 x + 1)$$

$$= \frac{\sin^2 x}{\cos^2 x} \left( \frac{1}{\cos^2 x} \right)$$

$$= \frac{\sin^2 x}{\cos^4 x}$$

$$= \text{Left side}$$

**b)** Verify for  $x = 30^\circ$ :

$$\text{Left side} = \cos 30^\circ + \cos 30^\circ \tan^2 30^\circ$$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6}$$

$$= \frac{2\sqrt{3}}{3}$$

$$\text{Right side} = \sec 30^\circ$$

$$= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3}$$

$$= \text{Left side}$$

Example:

$$\begin{aligned} \text{Left side} &= \cos x + \cos x \tan^2 x \\ &= \frac{\cos^3 x + \cos x \sin^2 x}{\cos^2 x} \end{aligned}$$

$$= \frac{\cos x (\cos^2 x + \sin^2 x)}{\cos^2 x}$$

$$\frac{1}{\cos x}$$

$$\text{Right side} = \sec x$$

$$= \frac{1}{\cos x}$$

$$= \text{Left side}$$

**9. a)** Verify for  $x = 3.2$ :

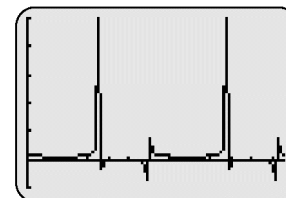
$$\begin{aligned} \text{Left side} &= \frac{\cos^2 3.2}{1 + 2\sin 3.2 - 3\sin^2 3.2} \\ &\approx 1.14153 \end{aligned}$$

$$\begin{aligned} \text{Right side} &= \frac{1 + \sin 3.2}{1 + 3\sin 3.2} \\ &\approx 1.14153 \end{aligned}$$

**b)**

```

Plot1 Plot2 Plot3
Y1=(cos(X))^2/(1
+2sin(X)-3(sin(X
))^2)
Y2=(1+sin(X))/(
1+3sin(X))
Y3=
Y4=
    
```



From the graph, the equation appears to be an identity.

**10. a)** Example:

$$\text{Left side} = \tan \theta$$

$$\begin{aligned} \text{Right side} &= \frac{1 - \cos 2\theta}{\sin 2\theta} \\ &= \frac{1 - (1 - 2\sin^2 \theta)}{2\sin \theta \cos \theta} \end{aligned}$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta$$

$$= \text{Left side}$$

$$\text{b) } \theta \neq \frac{n\pi}{2}; n \in \mathbb{I}$$



**11. Example:**

$$\begin{aligned} \text{Left side} &= 1 + \sin 2x \\ &= 1 + 2 \sin x \cos x \end{aligned}$$

$$\begin{aligned} \text{Right side} &= (\sin x + \cos x)^2 \\ &= \sin^2 x + 2 \sin x \cos x + \cos^2 x \\ &= 1 + 2 \sin x \cos x \\ &= \text{Left side} \end{aligned}$$

**12. Example:**

$$\begin{aligned} \text{Left side} &= \cos 3x + 1 \\ &= \cos(2x + x) + 1 \\ &= \cos 2x \cos x - \sin 2x \sin x + 1 \\ &= \cos x(2 \cos^2 x - 1) - 2 \sin x(\sin x \cos x) + 1 \\ &= 2 \cos^3 x - \cos x - 2 \sin^2 x \cos x + 1 \\ &= 2 \cos^3 x - \cos x - 2 \cos x(1 - \cos^2 x) + 1 \\ &= 4 \cos^3 x - 3 \cos x + 1 \end{aligned}$$

$$\begin{aligned} \text{Right side} &= 4 \cos^3 x - 3 \cos x + 1 \\ &= \text{Left side} \end{aligned}$$

**BLM 6-5 Section 6.4 Extra Practice**

1. a)  $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

b)  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \text{ and } \frac{7\pi}{4}$

c)  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \text{ and } \frac{7\pi}{4}$     d)  $\pi$

2. a)  $0^\circ, 72^\circ, 144^\circ, 216^\circ, 288^\circ$     b)  $270^\circ$

c)  $90^\circ, 270^\circ$

3. a)  $\frac{2\pi}{3}, \frac{4\pi}{3}$

b) no solution

c)  $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

4.  $-135^\circ, -45^\circ, 45^\circ, 135^\circ$

5.  $\frac{3\pi}{4}, \frac{7\pi}{4}$

6. The error was in dividing 1 by 2.

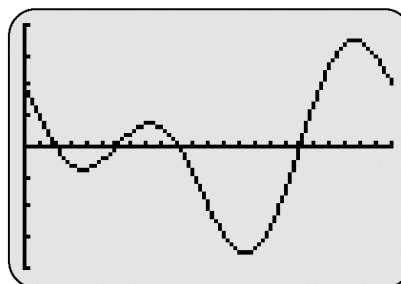
$$\begin{aligned} \sin 2x &= 1 \\ 2x &= 90^\circ \\ x &= 45^\circ \end{aligned}$$

7. a) The student used  $2\pi$  rather than  $\pi$ ; because the equation is  $\sin 2x = 1$ , the period of the function is  $\pi$ .

b)  $\frac{\pi}{4} + \pi n, \frac{5\pi}{4} + \pi n; n \in \mathbb{I}$

**8. a) Graph the function**

$Y_1 = \cos x - 2 \sin x \cos x$  using Xmin = 0 and Xmax =  $2\pi$ . The  $x$ -intercepts are the solutions.



b)  $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

9.  $\frac{\pi}{2} + 2\pi n, \frac{\pi}{4} + \pi n; n \in \mathbb{I}$

10.  $60^\circ, 120^\circ, 240^\circ, 300^\circ$

General Solution:  $60^\circ + 180^\circ n, 120^\circ + 180^\circ n; n \in \mathbb{I}$

**BLM 6-7 Chapter 6 Test**

1. A                    2. C

3. A                    4. B                    5. D

6.  $\frac{-\sqrt{6} - \sqrt{2}}{4}$

7. 0.23

8.  $150^\circ$

9.  $\frac{5}{13}$

10.  $\sin\left(\frac{5y}{2}\right)$

11. a) Left side =  $\sin\left(x + \frac{\pi}{2}\right)$   
 $= \sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$   
 $= \sin(\pi)$   
 $= 0$

Right side =  $\csc x - 1$

$$\begin{aligned} &= \csc \frac{\pi}{2} - 1 \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

b) No; it is not true for all permissible values of  $x$ .

12. a) Left side =  $\sin^2 x + \cos^4 x$   
 $= \sin^2 30^\circ + \cos^4 30^\circ$   
 $= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^4$   
 $= \frac{13}{16}$



$$\begin{aligned} \text{Right side} &= \cos^2 x + \sin^4 x \\ &= \cos^2 30^\circ + \sin^4 30^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^4 \\ &= \frac{13}{16} \end{aligned}$$

**b) Example:**

$$\begin{aligned} \text{Left side} &= \sin^2 x + \cos^4 x \\ &= \sin^2 x + (1 - \sin^2 x)^2 \\ &= \sin^2 x + 1 - 2\sin^2 x + \sin^4 x \\ &= 1 - \sin^2 x + \sin^4 x \\ &= \cos^2 x + \sin^4 x \\ &= \text{Right side} \end{aligned}$$

$$\text{Right side} = \cos^2 x + \sin^4 x$$

**13. a)**  $x \neq 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$

**b) Example:**

$$\begin{aligned} \text{Left side} &= \frac{\tan x + \sec x}{\cot x} \\ &= \left(\frac{\sin x}{\cos x} + \frac{1}{\cos x}\right) \div \cot x \\ &= \left(\frac{\sin x + 1}{\cos x}\right) \div \left(\frac{\cos x}{\sin x}\right) \\ &= \left(\frac{\sin x + 1}{\cos x}\right) \times \left(\frac{\sin x}{\cos x}\right) \\ &= \frac{(\sin x + 1)\sin x}{1 - \sin^2 x} \\ &= \frac{(\sin x + 1)\sin x}{(1 - \sin x)(1 + \sin x)} \\ &= \frac{\sin x}{(1 - \sin x)} \\ &= \text{Right side} \end{aligned}$$

$$\text{Right side} = \frac{\sin x}{1 - \sin x}$$

**14.**  $\pm \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

**15.**  $\frac{\pi}{3} + \pi n, \frac{2\pi}{3} + \pi n; n \in \mathbb{I}$

**BLM U2-2 Unit 2 Test**

1. A
2. D
3. A
4. A
5. B
6. B
7. D
8. 2.18 m
9.  $\frac{\sqrt{3} - 1}{2}$
10.  $\frac{\pi}{3} + \pi n, \frac{2\pi}{3} + \pi n, n \in \mathbb{I}$
11.  $y = 6\cos\frac{\pi}{5}(x - 3) + 4$
12. 2
13.  $B \neq 180^\circ n, n \in \mathbb{I}$
14.  $\tan 60^\circ; \sqrt{3}$
15. **a)** Period:  $4\pi$ ; phase shift: 4 left

**b)** The graph of  $f(\theta)$  is translated left  $\frac{9\pi}{2}$

**c)** -1.9

**16. a)** Left side =  $\frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$

$$\begin{aligned} &= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} \\ &= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{Right side} &= \cos(2 \times 30^\circ) \\ &= \cos 60^\circ \\ &= \frac{1}{2} \end{aligned}$$

Right side = Left side



$$\begin{aligned}\text{b) Left side} &= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \\ &= \left(1 - \frac{\sin^2 \theta}{\cos^2 \theta}\right) \div \sec^2 \theta \\ &= \left(\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}\right) \times \cos^2 \theta \\ &= \cos^2 \theta - \sin^2 \theta \\ &= \cos 2\theta\end{aligned}$$

Left side = Right side

$$\text{c) } x \neq \frac{\pi}{2} + \pi n, n \in \mathbb{I}$$

**17. a)** Amplitude: 2; range:  $1 \leq y \leq 5$

$$\text{b) } y = 2 \sin 3\left(x + \frac{\pi}{6}\right) + 3; y = 2 \cos 3x + 3$$

**18. a)**  $b = 3$

**b)** original graph is horizontally stretched by a factor of 6

**19. a)** Yes;

$$2 \sin^2 x - \sin x - 1 = (2 \sin x + 1)(\sin x - 1)$$

**b)**  $90^\circ, 210^\circ, 330^\circ$

