Section 6.3 Extra Practice

1. Rewrite each expression in terms of sine and cosine only. Then simplify.
   a) \( \frac{\sec x}{\tan x} \)
   b) \( \frac{\cot^2 x}{1 - \sin^2 x} \)
   c) \( \frac{\csc x - \sin x}{\cot x} \)

2. Factor and simplify each rational trigonometric expression.
   a) \( \frac{\tan x - \tan x \sin^2 x}{\cos^2 x} \)
   b) \( \frac{\sin^2 x + 
   \sin x - 6}{5 \sin x + 15} \)
   c) \( \frac{\cos^2 x - 4}{7 \cos x - 14} \)
   d) \( \frac{\sin^2 x \tan x - \tan x}{\sin x \tan x + \tan x} \)

3. Use the Pythagorean identities to prove each identity for all permissible values of \( x \).
   a) \( \csc^2 x (1 - \cos^2 x) = 1 \)
   b) \( (\tan x - 1)^2 = \sec^2 x - 2 \tan x \)
   c) \( \frac{\sin^2 x + \cos^2 x}{\sec x} = \cos x \)

4. Prove each identity. Use a common denominator to express two terms as one term, when necessary.
   a) \( \frac{1 + \tan x}{1 + \cot x} = \tan x \)
   b) \( \frac{\sec x - \sin x}{\sin x \cos x} = \cot x \)
   c) \( \frac{\cot x + \tan x}{\sec x} = \csc x \)

5. Prove each identity, using factoring.
   a) \( \frac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x \)
   b) \( \frac{\sin x + \tan x}{\cos x + 1} = \tan x \)
   c) \( \frac{\cos x + 1}{\sin x + \tan x} = \cot x \)

6. Verify each potential identity, then prove each identity.
   a) \( \frac{\cos x}{\sin x} = \frac{1 + \sin x}{\cos x} \)
   b) \( \frac{1 + \cos x}{\sin x} = \frac{\sin x}{1 - \cos x} \)
   c) \( \frac{\csc x}{\sec x - 1} + \frac{\cos x}{\sec x + 1} = 2 \cot^2 x \)

7. Prove the following algebraically.
   a) \( \cos (x + y) \cos (x - y) = \cos^2 x - \sin^2 y \)
   b) \( \frac{1 + \cos 2x}{\sin 2x} = \cot x \)
   c) \( 1 + \sin 2x = (\sin x + \cos x)^2 \)
   d) \( \sec^2 x = \frac{2}{1 + \cos 2x} \)

8. Verify each equation is true for \( x = 30^\circ \). Then prove each equation is an identity.
   a) \( \sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x \)
   b) \( \cos x + \cos x \tan^2 x = \sec x \)

9. Consider the equation
   \[ \frac{\cos^2 x}{1 + 2 \sin x - 3 \sin^2 x} = \frac{1 + \sin x}{1 + 3 \sin x} . \]
   a) Show that the equation is true for \( x = 3.2 \) radians.
   b) Use a graph to show that the equation may be an identity.

10. a) Prove that \( \tan \theta = \frac{1 - \cos 2\theta}{\sin 2\theta} . \)
     b) State any non-permissible values.

11. Prove the following identity.
    \( 1 + \sin 2x = (\sin x + \cos x)^2 \)

12. Prove the following identity.
    \( \cos 3x + 1 = 4 \cos^3 x - 3 \cos x + 1 \)
Section 6.4 Extra Practice

1. Solve each equation algebraically over the domain $0 \leq x < 2\pi$.
   a) $\sin 2x - \cos x = 0$
   b) $\cos 2x = 0$
   c) $2\cos^2 x - 1 = 0$
   d) $\cos^2 x - 2 = \cos x$

2. Solve each equation algebraically over the domain $0^\circ \leq x < 360^\circ$.
   a) $\cos 2x = \cos 3x$
   b) $2 \cos^2 x - 5 \sin x - 5 = 0$
   c) $\cot^2 x = 0$

3. Rewrite each equation in terms of cosine only. Then, solve algebraically for $0 \leq x < 2\pi$.
   a) $\cos 2x - 5 \cos x = 2$
   b) $\cot^2 x + 2 = 0$
   c) $1 + \cos x = 2 \sin^2 x$

4. Solve $2 \cos^2 x = 1$ algebraically over the domain $-180^\circ \leq x \leq 180^\circ$.

5. Solve $\tan^2 x + 2 \tan x + 1 = 0$ algebraically over the domain $0 \leq x < 2\pi$.

6. Determine the mistake the student made in the following work. Then, complete a correct solution.
   $$\sin 2x = 1$$
   $$\sin x = \frac{1}{2}$$
   $$x = 60^\circ \text{ and } 120^\circ$$

7. A student is asked to write the equation of the general solution of the following equation: $\sin 2x = 1$
   The solutions for this equation in the domain $0 \leq x < 2\pi$ are $\frac{\pi}{4}, \frac{5\pi}{4}$.
   The student writes the general solution as $\frac{\pi}{4} + 2\pi n, \frac{5\pi}{4} + 2\pi n; n \in \mathbb{Z}$.
   a) What error did the student make?
   b) Write the correct general solution for this equation.

8. a) Explain how to solve the equation $\cos x - 2 \sin x \cos x = 0$ graphically, using the intersection feature of the graphing calculator.
   b) Solve the equation from part a) algebraically over the domain $0 \leq x < 2\pi$.

9. Solve $(\sin x - 1)(\tan x - 1) = 0$ algebraically for all values of $x$.

10. Solve the following equation for $x$.
    Give the general solution in degrees.
    $2 \cos 2x + 1 = 0$