5.1 Exponents

Review of Exponential Rules

1. $b^0 = 1$ 2. $b^x \cdot b^y = b^{x+y}$ 3. $\frac{b^x}{b^y} = b^{x-y}$

4. $(b^x)^y = b^{xy}$ 5. $b^{-x} = \frac{1}{b^x}$ 6. $\left(\frac{b}{a}\right)^{-x} =$
$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

7. $(ab)^x = a^x b^x$ 8. $a^x = a^y$ if and only if $x = y$

Example 1:

Simplify: Make each a single term with an exponent

a.) $(4^x)^{2+x}\ (32^x)^{-x}$ b.) $\frac{9^x(27^{x-3})}{243^{x+1}}$

Example 2:

Solve: Change the base to be the same to solve

a.) $3^{3x+4} = 81^{x+2}$

b.) $8(8)^x = 2$

Graphing Exponential Functions

$y = b^x, b > 1$

$y = b^x, 0 < b < 1$

The point $(1, b)$ appears on both graphs:

Properties of Exponential Graphs

For graphs of the form $y = b^x$**,** $b > 0, b \neq 1$

Domain: $x \in R$

Range: $y > 0$

y-intercept = 1

Horizontal asymptote at y = 0

i.) When $0 < b < 1$, the graph is decreasing (decay)

ii.) When $b > 0$, the graph is increasing (growth)

Example 3: Sketch

$$y = 3^{x+1} + 2$$

Applications:

Exponential equations are found in a general form:

$P = P_0 b^x$ where P is the final amount

P_0 is the initial amount

b is the rate of growth or decay

A. Compound Interest:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = final

P = principle, or initial amount

r = rate of yearly interest

n = number of times yearly interest is compounded

t = time (in years)

Example 4:

How much more would you earn in two years if you compounded daily compared to monthly with an initial investment of $1000?

B. Growth and Decay Formulas:

$$A = A_0(b)^{\frac{t}{T}}$$

A = final

A_0 = initial amount

b = growth or decay value (e.g., half life use $\frac{1}{2}$)

T = time of growth or decay (e.g., half-life time)

t= total time

Example 5:

a. An element has a half-life of 30 years. If 1.0 mg of this element decays over a period of 90 years, how many mg of this element would remain?

b. An element has a half-life of 29 hours. How many mg of the element will remain after 46 hours?

Another form of growth and decay is written in the form

$$A = A_0 e^{kt}$$

This has many Calculus applications

e ≈ 2.71828

k = proportional constant