Ch 3.4: The Remainder and Factor Theorems:

The Remainder Theorem:

If the polynomial P(x) is divided by x-a, the remainder is P(a).

<u>Reason:</u>

From last chapter:

$$P(x) = (x - a)q(x) + r(x)$$

Example 1:

What is the remainder when $P(x) = 3x^4 - 5x^2 + 6x - 2$ is divided by $(x + 1)$?

Example 2:

For what value of *k* will the remainder be -2 when $P(x) = x^3 - 2x^2 + kx - 5$ is divided by $(x - 3)$?

Factor Theorem:

If a polynomial $P(x)$ is divided by $(x - a)$ and there is no remainder, then $(x - a)$ is a factor of $P(x)$.

In other words, if $P(a) = 0$, then $(x - a)$ is a factor of $P(x)$.

Example 3:
Does $P(x) = x^3 + 2x^2 + 4x + 8$ have a factor of $(x + 2)$?

Rational Root Theorem:

The possible roots of polynomial $P(x) = a_n x^n + \ldots + a_0$ are the possible factors of the constant divided by the possible factors of the leading terms:

ie

$$\frac{\text{Factors of } a_0}{\text{Factors of } a_n}$$

Example 4: Determine the possible roots

$2x^4 - 5x^3 - 12$ has possible roots:

Factoring Polynomials:
To factor a polynomial, we must use the rational root theorem and factor theorem to determine the roots.
Using the possible roots, we can try to find a root that satisfies the factor theorem; in other words, find a value of "*a*" such that $P(a) = 0$.

<u>Steps:</u> Factoring (or solving) an n>2 degree polynomial.

1. Find all possible roots
2. Find a root that satisfies $P(a) = 0$; start testing with smaller values.
3. Synthetically divide the factor.
4. Repeat until completely factored; do not forget any division steps.

Example 5: Factor completely:

a.) $x^3 - 2x^2 - 13x - 10$

b.) $2x^3 - 7x^2 - 7x + 12$

Example 6: Solve by factoring:

a.) $x^4 - 3x^3 + x^2 + 3x - 2 = 0$

b.) $x^4 + 4x^3 + 2x^2 - 5x - 2 = 0$

Note: Factors can always be repeated!

Example 7: Factor, then sketch
a.) $y = x^5 - 4x^4 + 14x^2 - 17x + 6$