Ch 3.1 Polynomials

Definition: A polynomial is given in the form:

$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0$

Where $a_n \in \mathrm{R}$, $n \in Z^+$

The **degree** is n and the **leading coefficient** is a_n

Examples of polynomials:

Polynomial	Degree	Leading Coefficient	Name
$f(x) = 3$			
$g(x) = \frac{2x - 5}{2}$			
$h(x) = -2x^2 + 4x - 5$			
$j(x) = 0.3x^3 + \sqrt{3}x + 2$			
$k(x) = -\sqrt{5}x^3 + 4x - 5$			

Examples of non-polynomials:

Non-Polynomial	Reason
$f(x) = 3x^{-2}$	
$g(x) = 2\sqrt{x}$	
$h(x) = -2x^{0.5} - 5$	
$j(x) = \frac{2x^2 - 5x}{x}$	
$k(x) = \sqrt{-5}x^3 - 5$	

Properties of Polynomials:

<u>Constant Value:</u>

The constant value is the y-value when $x = 0$ (Also known as the y-intercept)

Example 1: Find the constant value (y-intercept)

a.) $f(x) = -2x^2 - 5x + 6$

b.) $g(x) = -2(x-2)(x-6)(x+3)$

c.) $h(x) = (3-x)^3(x-1)^2$

<u>Zeros of Polynomials</u>

The zeroes of a function occur when the graph crosses the x-axis. (also known as the root)

Find the roots by factoring.

Example 2: Find the zeroes of:

a.) $f(x) = -16x^4 + 64x^2$

b.) $(x) = -2(x-2)(x-6)(x+3)$

c.) $(x) = 2x^2 + 9x - 5$

d.) $g(x) = x^3 - 5x^2 - 12x + 60$

Turning Points

A **turning point** is also known as a local or absolute maximum or minimum.
A polynomial function has at most *n* - 1 turning points where *n* is the degree.

Multiplicity

The **multiplicity** of a function is the number of times a zero is repeated:

Example: Find the degree, and the multiplicity of each zero

a.) $f(x) = (x + 4)(x - 5)^2(x + 2)^5$

b.) $f(x) = x(x + a)^3(x - b)^4(x + c)^2$

Graphs of Polynomial:

Continuous:

A function is continuous if there are no jumps or holes. All polynomials are continuous.

Smooth:

Smooth functions have no sharp edges. All polynomials are smooth.

End Behaviour:

The **end behaviour** of a polynomial function is the y-value of the function as the x-value approaches +∞ and -∞.

The end behaviour is based on the leading term of the polynomial; specifically the degree and whether the leading coefficient is positive or negative.

1. Constant functions: $f(x) = c$, where c is a constant

As $x \to \pm\infty, f(x) \to c$

Example:

2. Odd Degree functions: $f(x) = ax^n$, where n is an odd integer:

If $a > 0$, as $x \to \infty, f(x) \to \infty$

as $x \to -\infty, f(x) \to -\infty$

Examples: $f(x) = x^3 + \ldots$, $f(x) = x^5 + \ldots$

If $a < 0$, as $x \to \infty, f(x) \to -\infty$

as $x \to -\infty, f(x) \to \infty$

Examples: $f(x) = -x^3 + \ldots$, $f(x) = -x^5 + \ldots$

2. Even Degree functions: $f(x) = ax^n$, where n is an even integer:

If $a > 0$, as $x \to \infty, f(x) \to \infty$

as $x \to -\infty, f(x) \to \infty$

Examples: $f(x) = x^2 + \ldots$, $f(x) = x^4 + \ldots$

If $a < 0$, as $x \to \infty, f(x) \to -\infty$

as $x \to -\infty, f(x) \to -\infty$

Examples: $f(x) = -x^2 + \ldots$, $f(x) = -x^4 + \ldots$