Ch 2.6 Combined Transformations

$y = f(x)$ becomes $y = af\big(b(x - c)\big) + d$

Vertical Transformations (Affecting y-values)

- $|a| > 1$ Vertical expansion
    - (m, n) ==> (m, a×n)
- $|a| < 1$ Vertical compression
    - (m, n) ==> (m, a×n)
- $a < 0$ Reflection across x-axis
    - (m, n) ==> (m, a×n)
- $d > 0$ Translation up
    - (m, n) ==> (m, n + d)
- $d < 0$ Translation down
    - (m, n) ==> (m, n + d) [note d is negative]

Horizontal Transformations (Affecting x-values)

- $|b| > 1$ Horizontal compression
    - (m, n) ==> ($\frac{1}{b}$m, n)
- $|b| < 1$ Horizontal Expansion
    - (m, n) ==> ($\frac{1}{b}$m, n)
- $b < 0$ Reflection across y-axis
    - (m, n) ==> ($\frac{1}{b}$m, n)
- $c > 0$ Translation right
    - (m, n) ==> (m + c, n)
- $c < 0$ Translation left
    - (m, n) ==> (m + c, n) [note c is negative]

Order of Transformations:

1. Inverse

2. Compression/Expansion/Reflection

3. Translations

Example 1:

$y = f(x)$ is transformed to

$y = 3f(-2(x + 4)) - 5$

a.) State the transformations

b.) The point (12, -9) becomes

Example 2:

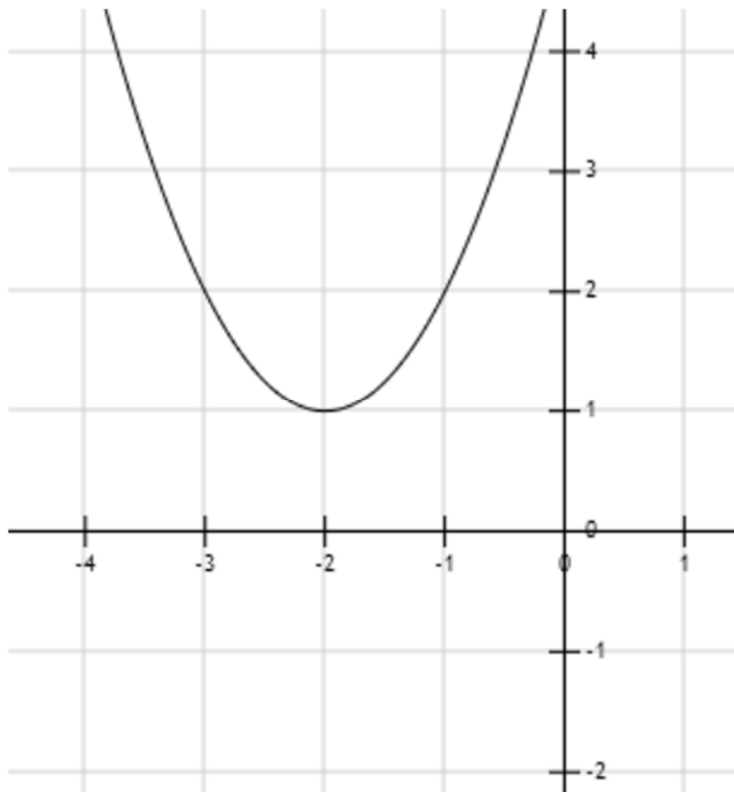$y = f(x)$ is transformed to

$y + 5 = f(2x - 4)$

a.) State the transformations

b.) The point (12, -9) becomes:

More Examples:

1. If the point (4, 1) is on the graph $y = f(x)$, what point is on $y = -2f(\frac{1}{3}(x - 1)) + 4$

2. If the point (4, 3) is on the graph $y = f(x)$, what point is on $y = 2f(4 - x) + 3$

3. If the point (4, 1) is on the graph $y = f(x)$, what point is on $y = 0.5f^{-1}(2x + 5) + 4$

Example 3:

Graph the transformation of $y = f(x)$ to $y = 2f(x) - 2$

General Examples:

1. Given: $y = x^2 + x$, write the equation after:

a.) Translation up 5, right 2

b.) Horizontal Compression by a factor of 2 (or to a factor of $\frac{1}{2}$) then right 4 and up 1

2. Given: $x^2 + y^2 = 1$ write the equation after:

a.) Translation down 3, left 2

b.) Vertical Expansion by a factor of 3, horizontal compression by 2, down 1 and left 2