3.4 Velocity and Other Rates of Change

Motion Along a Line

Given an object is moving along a line with its position given as

$$s = f(t)$$

Average velocity:

$$v_{av} = \frac{displacement}{time} = \frac{\Delta s}{\Delta t} = \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

(assume you are going in one direction)

Suppose you drive your car for 2 h and travel exactly 80 mi. What is your **average velocity**?

Instantaneous velocity = first derivative of the distance function =

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

In the Calculus AP course, we are just concerned with rectilinear motion (motion of a particle along a line)

Summary	s(t)	gives position	s'(t) =0 _____
	s'(t)	gives velocity	s'(t) > 0_____
			s'(t) < 0_____

Example 1 $s(t) = t^3 - 6t^2 + 9t$ (t is measured in sec, s is measured in ft)

a) find the velocity at time t

b) what is the velocity after 2 sec, after 4 sec?

c) when is the particle at rest?

d) when is the particle moving to the right (in a positive direction)?

e) draw a diagram to represent the position of the particle for $t \geq 0$ on a number line. Then find its velocity at each time.

t	s	v
0		
1		
2		
3		
4		
5		

f) (distance / time graph)

g) What is the average velocity during the first 5 seconds?

h) What is the displacement during the first 5 seconds?

i.) What is the **total** distance traveled from time t = 0 to t = 5 sec? (Different than h)

Sometimes you can consider rectilinear motion of a vertical line (then right is up and left is down)

Example 2 The vertical height s (in feet) of a ball thrown upwards from a tall building is described by the position function: $s(t) = -16t^2 + 96t + 640$ (t in sec)

a) What is the initial velocity at $t = 0$

b) What is the height of the ball when the velocity = 0

c) What is the velocity of the ball when it returns to earth?

Speed = the absolute value of velocity speed = $|v(t)| = \left|\frac{ds}{dt}\right|$

Acceleration = the rate of change of the velocity = v'(t) = s''(t)

Example 3 A dynamic blast propels a heavy rock straight up with a launch velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ ft after *t* seconds.

a) How high does the rock go?

b) What is the velocity and speed of the rock when it is 256 ft above the ground on the way up? on the way down?

c) What is the acceleration of the rock at any time t during its flight (after the blast)?

d) When does the rock hit the ground?

Example 4 The position function of a particle moving on a line is $s(t) = t^3 + 4t^2 - 3t$ (t in sec, s in ft)

a) What is the velocity at $t = 1$

b) What is the acceleration at $t = 1$

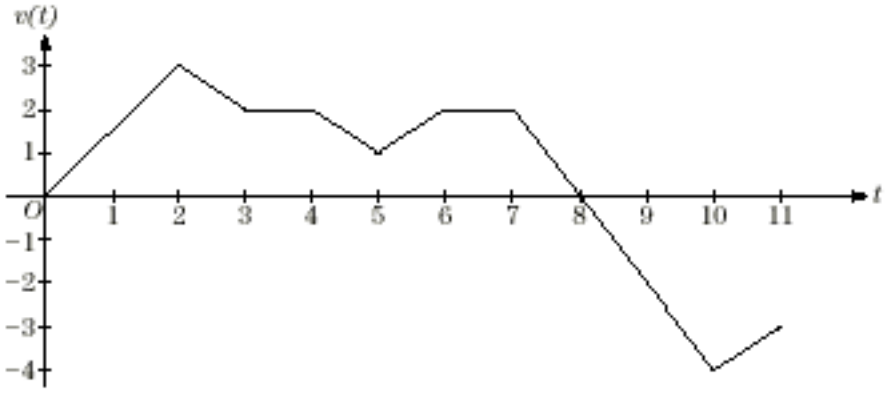
c) When is the particle at rest?

Example 5 Consider the position function $s(t) = t^3 - 6t^2$
Find v(t) and a(t) and graph s(t), v(t) and a(t) them over the interval $0 < t < 8$ use vertical scale between –40 and 60

Summary	$v > 0$	(moving right)
	$v < 0$	(moving left)
	$v = 0$	(particle stopped)

Example 6 Reading a Velocity Graph

A bug is crawling along a straight wire. The velocity, *v(t)*, of the bug at time *t*, $0 \le t \le 11$, is given in the graph below.



According to the graph,

a) at what time t does the bug change direction?

b) at what time t is the speed of the bug greatest?

c) when is the bug moving right? Moving left?

Example 6 Enlarging Circles

a) Find the instantaneous rate of change of the area *A* of a circle with respect to its radius *r*.

b) Evaluate the rate of change of *A* at $r = 5$ and $r = 10$.

c) If r is measured in inches and A is measured in square inches, what units would be appropriate for $\frac{dA}{dr}$?