3.3 Rules for Differentiation

Basic Rules

If *u* and *v* are differentiable functions of *x* and *c* is a constant, then:

Derivative of a Constant $\frac{d}{dx}c =$

Power Rule $\frac{d}{dx}(x^n) =$

Constant Multiple Rule $\frac{d}{dx}(cu) =$

Sum and Difference Rule $\frac{d}{dx}(u \pm v)$

Example 1 Differentiating using the basic rules

Find $\frac{dy}{dx}$ if

a.) $y = x^4 - 5x^2 + \frac{5}{4}x + 15$

b.) $y = \frac{x^4}{4} - \frac{5x^2}{7} + \frac{5}{4}x^\pi$

c.) $y = \frac{3}{x^3} - \frac{1}{x} + 3$

d.) $y = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x^3}$

e.) $y = \frac{5x + x^5}{2x^2}$

Example 2 Finding Tangent Lines

Use the results from part e) to find the equation of the tangent to the curve $y = \frac{5x+x^5}{2x^2}$ at the point (1,3). Support your answer graphically.

Example 3 Finding Horizontal Tangents

a) Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so, where?

b) Determine the x-values where the curve $y = 0.2x^4 - 0.7x^3 - 2x^2 + 5x + 4$ has horizontal tangents.

More Differentiation Rules

The Product Rule $\frac{d}{dx}(uv) =$

The Quotient Rule $\frac{d}{dx}\left(\frac{u}{v}\right) =$

Example 4 Differentiating a Product

Find $f'(x)$ if $f(x) = (x^2 + 1)(x^3 + 2)$

Example 5 Differentiating a Quotient

Find $f'(x)$ if $f(x) = \frac{x^2-1}{x^2+1}$

Example 6 Working With Numerical Values

Let $y = uv$ be the product of the functions u and v. Find $y'(2)$ if

$$u(2) = 3, u'(2) = -4, v(2) = 1, and\ v'(2) = 2$$

Example 7 Second and Higher Order Derivatives

Find the first four derivatives of $y = x^5 - x^3 + 2$