3.2 Differentiability

How $f'(a)$ Might Fail to Exist

A function will not have a derivative at a point $P(a, f(a))$ where the slopes of the secant lines, $\frac{f(x)-f(a)}{x-a}$ fail to approach a limit as x approaches a.
In other words, a function is differentiable if at a point *a* the function exists, is continuous, and the derivative $f'(a)$ exists

1. A *corner* (where the one-sides derivatives differ)

$$y = |x|$$

2. A *cusp* (where the slopes of the secant lines approach ∞ from one side and $-\infty$ from the other)

$$y = x^{\frac{2}{3}}$$

3. A *vertical tangent* (where the slopes of the secant lines approach $-\infty$ or ∞ from both sides)

$$y = x^{\frac{1}{3}}$$

4. A *discontinuity* (which will cause one or both of the one-sided derivatives to be nonexistent)

$$y = \begin{cases} -1, x < 0 \\ 1, x \geq 0 \end{cases}$$

Example 1 Finding Where a Function is Not Differentiable

Find all points in the domain of $f(x) = 2|x + 3| - 5$ where *f* is not differentiable.

Theorem: Differentiability Implies Continuity

If *f* has a derivative at $x = a$, then *f* is continuous at $x = a$.

Be careful: Differentiability implies continuity but continuity does not necessarily imply differentiability. Why not?