3.1 Derivative of a Function

Definition of the Derivative with respect to *x*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This finds the function of the slope at any point on the graph (if the slope exists)

Example 1 Applying the Definition

Differentiate (that is, find the derivative of) $f(x) = x^3 - x^2$

Alternate Definition of the Derivative at a Point

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Example 2 Applying the Alternate Definition

Differentiate $f(x) = \sqrt{x+1}$ using the alternate definition at $x = 3$

Notation

$f'(x)$: f prime of x

$\frac{dy}{dx}$: dy dx

y'

Example 3 Recognizing a given limit as a derivative

Given $f'(x)$, determine $f(x)$.

a.) $\lim\limits_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

b.) $\lim\limits_{h \to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h}$

c.) $\lim\limits_{h \to 0} \frac{\frac{x+h}{(x+h)^2+1} - \frac{x}{x^2+1}}{h}$

d.) $\lim\limits_{h \to 0} \frac{e^{x+h} - e^{x}}{h}$

One-Sided Derivatives

The Right-hand derivative at *a*

$$\lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h}$$

The Left-hand derivative at *b*

$$\lim_{h\to 0^-} \frac{f(b+h)-f(b)}{h}$$

Example 4 One-Sided Derivatives Can Differ at a Point

Show that the following function has left-hand and right-hand derivatives at x=3, but no derivative there.

$$y = \begin{cases} x^2, & x < 3 \\ 2x, & x \geq 3 \end{cases}$$