9.2 Analysing Rational Functions

Non-permissible values for rational functions are located for all x-values that make the denominator equal to 0. The two types of non-permissible values that we will look at are:

1. Vertical asymptotes: the graph approaches large y-values as the x-values approach the vertical asymptote

2.Points of discontinuity (hole): an ordered pair (x, y) where the graph is not continuous; results in a single point missing from the graph (represented by an open circle).

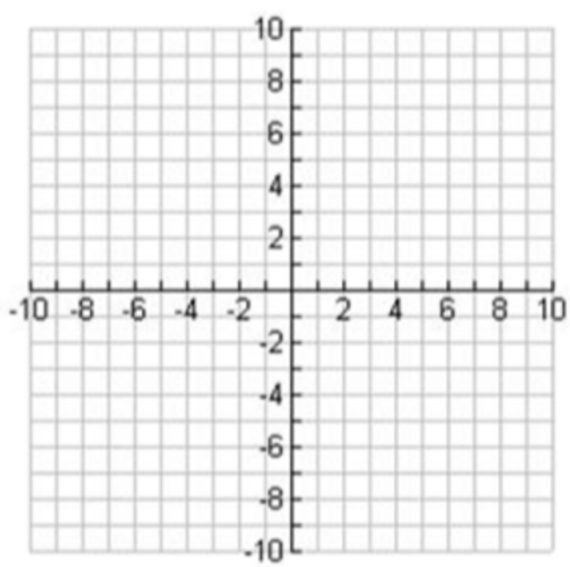
-occurs when the rational function is simplified by dividing the numerator and denominator by a common factor that involves a variable

Example 1: Graph the following rational function. Analyze the behavior near the non-permissible value

a.)

$$f(x) = \frac{x^2 - 5x - 6}{x + 1}$$

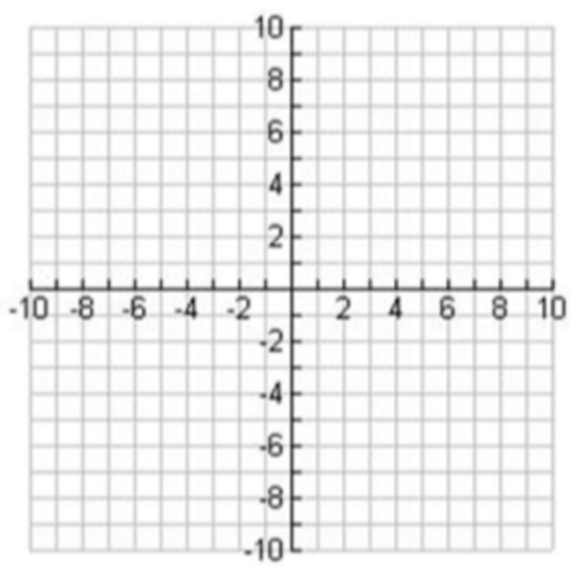
Characteristic	Equation: $f(x) = \frac{x^2 - 5x - 6}{x + 1}$
Non-Permissible value	
Behaviour Near each non-permissible value	
Domain	
Range	



b.)

$$g(x) = \frac{x^2 + 2x}{4 + 2x}$$

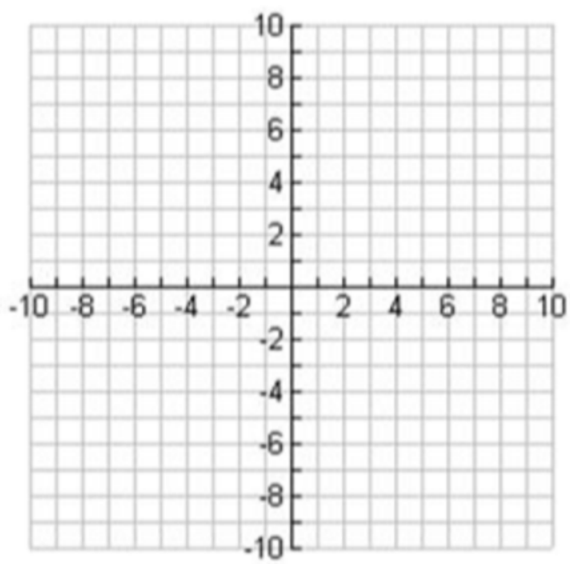
Characteristic	Equation: $g(x) = \frac{x^2 + 2x}{4 + 2x}$
Non-Permissible value	
Behaviour Near each non-permissible value	
Domain	
Range	



c.)

$$h(x) = \frac{x^2 + 5x - 6}{x^2 - 2x + 1}$$

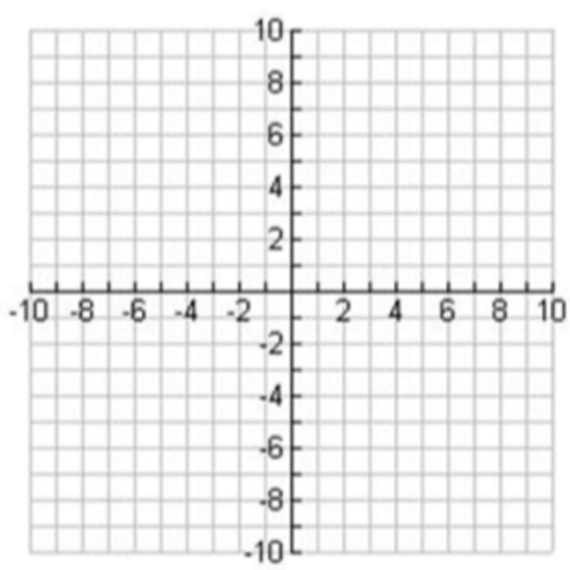
Characteristic	Equation: $h(x) = \frac{x^2 + 5x - 6}{x^2 - 2x + 1}$
Non-Permissible value	
Behaviour Near each non-permissible value	
Domain	
Range	



d.)

$$k(x) = \frac{x^2 - 9}{x^2 + 5x + 6}$$

Characteristic	Equation: $k(x) = \frac{x^2 - 9}{x^2 + 5x + 6}$
Non-Permissible value	
Behaviour Near each non-permissible value	
Domain	
Range	



Horizontal Asymptotes of Rational Functions:

Given $f(x) = \frac{g(x)}{h(x)}$ and m is the degree of the numerator (top) and n is the degree of the denominator (bottom). In other words:

$$f(x) = \frac{g(x)}{h(x)} = \frac{ax^m + \cdots + a_0}{bx^n + \cdots + b_0}$$

1. If $m < n$, then y = 0 is a horizontal asymptote.

2. If $m = n$, then $y = \frac{a}{b}$ (ratio of the leading coefficients) is the horizontal asymptote.

3. If $m > n$, then there is a slant or oblique asymptote (which is not covered in this course).

Example 2: Determine the horizontal asymptote of the rational functions:

a.) $f(x) = \frac{4x-6}{3x^2+4x-2}$

b.) $g(x) = \frac{6-x^2}{3x^2-4x+4}$

c.) $h(x) = \frac{x^3-3x-2}{3x^2}$

Intercepts of Rational Functions:

To determine the y-intercept of a rational function, set the x-value to 0.

To determine the x-intercept of a function, set the y-value to 0; in general, we can set the numerator to 0 after factoring and reducing to solve for the x-intercepts.

Example 2: Determine the x and y-intercepts:

a.) $y = \frac{4x^2-6x-4}{x^2+x-20}$

b.) $y = \frac{x^2+x-2}{x^2+19x-20}$

Example 3:

Determine the equation of the following rational functions: