8.4 Logarithmic and Exponential Equations

Note:

You can apply an exponent to both sides of an equation to eliminate a logarithm:

\[ \text{ie.} \]

\[ \log_a x = b \]

Example 1: Solve

a.) \( \log_7 x + \log_7 4 = \log_7 12 \)

b.) \( \log_2 (x - 6) = 3 - \log_2 (x - 4) \)
c.) \( \log_3(x^2 - 8x)^5 = 10 \)

d.) \( \log_3 x + \log_3(x + 6) = 3 \)

e.) \( 2\log_3 x + \log_3(x - 1) = 1 + \log_3 2x \)
You can also apply a logarithm to both sides of the equation (and apply the log rules) to solve when the variable is in the exponent.

**Example 2: Solve to 2 decimal places**

a.) \(4^x = 605\)

b.) \(5^{x-3} = 1700\)

c.) \(6^{3x+1} = 8^{x+3}\)
d.) \(3^x \cdot 2^{x-1} = 4\)

e.) \(5^x = 2 \cdot 3^x\)

**Example 3: Algebraically solve:**

The half-life of C-14 is 5730 years. A 1000 g bone containing C-14 decayed to 350 g of C-14. After how long did this occur?
**pH scales**

Example 4: The pH scale is a base 10 scale that measures the acidity (0-7) or alkalinity (7-14) of a solution with 7 being neutral water.

a.) The average pH from the Vancouver reservoirs is 6.8 on the pH scale. The average pH in Toronto is 7.5. How much more basic (more alkaline) is the pH in Toronto than in Vancouver?

b.) Tomato juice is 630 times more acidic than water. What is its pH reading?

**Earthquakes**

Example 5: The intensity of earthquakes are measured on a Richter scale of base 10.

a.) If an earthquake in Chile is 250 times more intense than an earthquake measuring 4.6 on the Richter scale, what is the reading in Chile?

b.) An earthquake in Japan measured 7.8 on the Richter scale. A smaller earthquake off the coast of BC measured 5.6. How much more intense as Japan's earthquake?