7.3 Solving Exponential Functions:

Review of Exponential Rules

1. \( b^0 = 1 \)  
2. \( b^x \cdot b^y = b^{x+y} \)  
3. \( \frac{b^x}{b^y} = b^{x-y} \)

4. \( (b^x)^y = b^{xy} \)  
5. \( b^{-x} = \frac{1}{b^x} \)  
6. \( \left(\frac{b}{a}\right)^x = \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \)

7. \( (ab)^x = a^xb^x \)  
8. \( a^x = a^y \) if and only if \( x = y \)

Example 1: Write each expression as a power with base 2:

a.) \( 4^3 \)

b.) \( \frac{1}{8} \)

c.) \( 2^3(\sqrt{16})^3 \)

Example 2: Simplify by changing the base

a.) \( (4^x)^{2^x} (32^x)^{-x} \)

b.) \( \frac{9^x(27^{x-3})}{243^{x+1}} \)
Solving Exponential Equations

Exponential Equations can be solved if the bases are the same on both sides. Otherwise, a graphing calculator can be used.

Example 3: Solve

a.) $4^{x+2} = 64^x$

b.) $3^{3x+4} = 81^{x+2}$

c.) $4^{x+2} = 8^{2x-3}$

d.) $8(8)^x = 2$
Example 4:

a.) (Algebraically) A colony of 20000 ants grows by 50% every hour. After how many hours will it take the colony to reach 151875 ants?

b.) (Graphing calculator) Determine how long $1000 needs to be invested in an account that earns 8.3% compounded semi-annually (once every 6 months) before it increases to $1500.