6.1 Reciprocal, Quotient and Pythagorean Identities

Note:
$\sin^2 x = (\sin x)^2$

Reciprocal Identities:

$$csc\ \theta = \frac{1}{\sin\theta}$$

$$sec\ \theta = \frac{1}{\cos\theta}$$

$$cot\ \theta = \frac{1}{\tan\theta}$$

An identity is a statement that is true for all possible values of the variable

Quotient Identities:

$$tan\ \theta = \frac{\sin\theta}{\cos\theta}$$

$$cot\ \theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Strategies for Simplifying a Trigonometric Expression:

1. Change all terms to *sine* or *cosine*

$$\frac{\cot x}{\csc x}$$

2. Simplify fractions; create one single fraction

$$a.)\ \frac{x+\frac{a}{x}}{x-\frac{a}{x}}$$

$$b.)\ \frac{\frac{\cos x}{sinx}+1}{\frac{1}{sinx}-1}$$

3. Factor:

a.) difference of squares

$1-\cos^4 x$

b.) Greatest common difference (GCF)

$\cos x-\cos^2 x\sin x$

4. Conjugate (multiply by the complement)

$$\frac{1}{1 - \sin x}$$

Example 1:

Simplify:

a.) $\frac{\sin^2 x}{\cos^2 x} + 1$

b.) $\sin x + \cos x \cot x$

c.) $\sin^2 x + \sin^2 x \cot^2 x$

d.) $\frac{\sin x + \tan x}{\csc x + \cot x}$

e.) $\frac{\cos^2 x}{1 + \sin x}$

Restrictions:

A trigonometric expression cannot have a zero in the denominator:

Example 2:

Determine the restriction on:

a.) $\tan x + \csc x$

b.) $\frac{\sec x}{\cot x}$