4.5 Newton's Method

LINEARIZATION – For any differentiable function, f, you can zoom in closer and closer to a point $x = a$ and the curve looks more and more like a line. Extend this line and it is the tangent line at $x = a$.

The tangent line through $(a, f(a))$ closely approximates the graph of $f(x)$ for the values of x near "a".

Linearization If f is a differentiable function at $x = a$, the approximating function $$L(x) = f(a) + f'(a)(x - a)$$ is the linearization of f at $x = a$ (Derived from slope formula)

Example 1:
Find the linear approximation of $\sqrt{1 + x}$ at $x = 0$. Use this to find an approximate value for $\sqrt{1.2}$

NEWTON'S METHOD is an example of local linearization. It's a method of approximating the zeros of a function and it converges very rapidly to an accurate approximation of the roots.

Process

1. Choose an x value, x_1 that you believe is close to the root (can check by estimation)
2. Draw a tangent line
3. the x intercept of the tangent becomes x_2
4. Repeat these steps (iteration) until the desired accuracy is reached

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Note:
Recursive formula – each new term depends on the previous one(s)
Iteration – set of steps that happen over and over again

Example 2:
Use Newton's Method to solve $x^3 + x + 1 = 0$ Start at $x_1 = 1$ and do 6 iterations (4 decimal places)