4.3 Connecting *f'* and *f''* with the Graph of *f*

Example 1: Use the first derivative test to find all the local and absolute extrema. Identify the intervals on which f is increasing and decreasing.

a.) $f(x) = x^3 - 12x - 5$

b.) $f(x) = (x^2 - 3)e^x$

Concavity:

The graph concaves up if y' is increasing on an interval:

The graph concaves down if y' is decreasing on an interval:

Inflection Point:

A point where the function has a tangent line and where the concavity changes.

For the graph of $y = f(x)$: When

a.) $y'' > 0$ the graph is concaving up

b.) $y'' < 0$ the graph is concaving down

c.) $y'' = 0$ there may be an inflection point

Example 3: A particle is moving along a horizontal line with the position:

$$s(t) = 2t^3 - 14t^2 + 22t - 5, t \geq 0$$

a.) Find the velocity and acceleration.

b.) Determine the concavity of the function.

Example 3: Determine the concavity and possible inflection points for the function:

$$f(x) = x^3 - 3x - 24x + 5$$

Second derivative Test for Local Extrema

1. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$

2. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$

Example 4 Find the extreme values of $f(x) = x^3 - 12x - 5$

Example 5: Sketch a possible graph of f given $f'(x) = 4x^3 - 12x^2$