4.2 Mean Value Theorem

MEAN VALUE THEOREM

Let f be differentiable on (a, b) and continuous on [a, b]. Then there is at least one point c in (a, b) where

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

In other words:

There is a tangent line between a and b at *c* where the slope is parallel to the slope of the secant line from *a* to *b*.

Graphically:

Example 1:
Let $f(x) = x^3 + 1$. Show that $f(x)$ satisfies the hypotheses of the Mean Value Theorem on the interval $[1, 2]$ and find all values *c* in this interval whose existence is guaranteed by the theorem.

Rolle's Theorem

If $f(x)$ meets these conditions

1) continuous on $[a, b]$
2) differentiable on (a, b)
3) $f(a) = f(b)$

Then, there is a point c on the interval $[a, b]$where $f'(x) = 0$

Increasing and Decreasing Functions

Let f be continuous on $[a, b]$ and differentiable on (a, b)

If $f'(x) > 0$ at each point of (a, b) then f increases on $[a, b]$

Graphically:

If $f'(x) < 0$ at each point of (a, b) then f decreases on $[a, b]$

Graphically:

First Derivative Test for **Local** Extrema

Let $f'(c) = 0$ or $f'(c)$ is **undefined**, then

1) Local Maximum occurs where the sign of $f'(x)$ changes from positive to negative at $x = c$

2) Local minimum occurs where the sign of $f'(x)$ changes from negative to positive at $x = c$

You must discuss the sign change of the slope when giving answers for local extrema.

Example 2:
Find the local extrema for $g(x) = x^4 - 4x^3 - 8x^2 - 1$

Example 3: Use a sign chart to determine the local max/mins

$g'(x) = \frac{(2x-3)^2(x+3)(x-1)}{x+6}$