4.1 Angles and Angle Measures

Recall:

Angles in standard position (on a coordinate plane):

Degree Measures:

One rotation is $360°$. Angles measured in the **counter clockwise** direction are **positive.** Angles measured in the **clockwise** direction are **negative.**

Co-terminal Angles:

Angles with the same terminal arm are co-terminal: they differ by a factor of $360°$.

ie. an angle, θ_1 is co-terminal with θ_2 if they differ by a factor of $360°$.
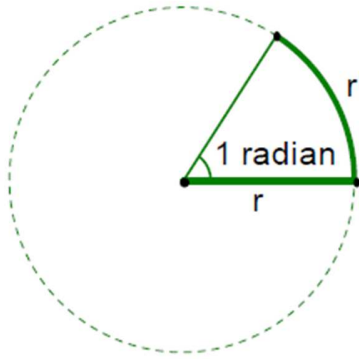
Example 1: Find a co-terminal angle, θ: $0 \leq \theta < 360°$

a.) $1850°$ b.) $- 2000°$

Radian Measure:

An arc of a circle with the same length as the radius of that circle corresponds to an angle of 1 radian. A full circle corresponds to an angle of 2π radians.

1 rotation is 360º OR 2π radians:

half a rotation is 180º or π radians:

Conversion from Degrees to Radians:

$$degrees \times \frac{\pi}{180°} = radians$$

Conversion from Radian to Degrees:

$$radians \times \frac{180°}{\pi} = degrees$$

Note: You can cancel out units to remember which equation to use.

Example 2:

i. Convert from radians to degrees:

a.) $\frac{3\pi}{2}$

b.) $\frac{11\pi}{6}$

c.) 2.5

ii. Convert from degrees to radians:

a.) 270°

b.) 120°

c.) 348°

Note: the answer is understood to be in radians if no symbol is placed after the numerical value.

Try:

Convert from radians to degrees or degrees to radians:

a.) 60°   b.) 30°

c.) $\frac{3\pi}{4}$ d.) 35

In Standard Position

Coterminal Angles in Radians

Two angles are coterminal if they differ by 360° **OR** _____ in radians.

θ_1 is co-terminal with θ_2 if:

$\theta_1 = \theta_2 + 360° \cdot n,$ **OR**

$\theta_1 = \theta_2 +$ _____ $\cdot n,$ where $n \in Z$

Example 3: Determine all co-terminal angles to θ in the given ranges, and state the quadrant where the terminal arm lies

a.) $\theta = -\frac{9\pi}{4}, 0 \leq \theta < 4\pi$

b.) $\theta = \frac{19\pi}{6}, -4\pi \leq \theta < -2\pi$

c.) $\theta = -\frac{7\pi}{3}$, $-2\pi \leq \theta < 2\pi$

Try: Find the co-terminal angles and the quadrants:

a.) $\theta = -\frac{5\pi}{4}$, $0 \leq \theta < 2\pi$

b.) $\theta = \frac{11\pi}{2}$, $-4\pi \leq \theta < 0$

c.) $\theta = \frac{\pi}{6}$

Arc Length

$a = r\theta$, θ is in radians

Example 4:

a.) Determine the arc length of a circle with radius 10cm and central angle of $\frac{\pi}{3}$

b.) Determine the arc length of a circle with diameter 12m and central angle of 40°

Try:

What is the degree measure of an angle θ opposite an arc of 25 m in a circle of diameter 20?