

3.9 Derivatives of Exponential and Logarithmic Functions

A logarithm is an exponent

$$\log_a x = y \quad a^y = x$$

Laws of Logarithms

1.) $\log_a nm = \log_a n + \log_a m$

2.) $\log_a \left(\frac{n}{m}\right) = \log_a n - \log_a m$

3.) $\log_a n^m = m \log_a n$

4.) $\log_a a^x = x$

5.) $a^{\log_a x} = x$

6.) $\log_a 1 = \log_a a^0 = 0$

Natural Logs

$\log_e x = \ln x$ is the natural log

Derivative of $y = \ln x$

Example 3

Find the derivative of each function:

a) $y = \ln x^2$

b) $y = \ln\left(\frac{3}{x}\right)$

c) $y = \ln(3x + 3)$

d) $y = \ln(\sin x)$

e) $y = (\ln x)^2$

f) $y = x \ln x$

g) $y = \ln \ln x$

Derivative of $y = a^x$

Derivative of $y = \log_a x$

Example 4: Derive

$$y = \frac{(x + 3)^5 \cdot \sqrt[8]{x + 6} \cdot (x^2 - 9)^{40}}{(9x^2 + 6)^2}$$