3.8 Derivatives of Inverse Functions and Inverse Trigonometric Functions

Example 1: Using implicit differentiation.

Let $f(x) = x^5 + 2x - 1$.

a) Since the point (1,2) is on the graph of *f*, (2, 1) is on the graph of f^{-1}.

b) Determine $\frac{df^{-1}}{dx}(2)$

Derivative of Inverse Functions:

$$\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'[f^{-1}(x)]}$$

(note: this only works if the inverse exists)

Example 2

Let $f(x) = x^3 + x^2 + 1$

a) $f'(x)$

b) Find $f^{-1}(3)$ and $\frac{d}{dx}(f^{-1}(3))$

Derivative of the Arcsine Function

Consider the function $y = \sin x$

What is the inverse of this function:

What is the domain and range (sketch)?

Determine the derivative of the inverse sine function:

$$\boxed{\frac{d}{dx}(\arcsin x) = \frac{d}{dx}(\sin^{-1} x) =}$$

Example 3

Determine: $\frac{d}{dx}(\sin^{-1} x^2)$

Derivative of the Arctangent Function

Consider the function $y = \tan x$

What is the inverse of this function:

What is the domain and range?

Determine the derivative of the inverse tangent function:

$\frac{d}{dx}(\arctan x) = \frac{d}{dx}\tan^{-1} x$

Example 4

A particle moves along a line so that its position at any time $t \geq 0$ is $s(t) = \tan^{-1}(\sqrt{t})$. What is the velocity of the particle when $t = 16$?

Example 5

a) Find an equation for the line tangent to the graph of $y = \tan x$ at the point $\left(-\frac{\pi}{4}, -1\right)$.

b) Find an equation for the line tangent to the graph of $y = \tan^{-1} x$ at the point $\left(-1, -\frac{\pi}{4}\right)$

Example 6

Find $\frac{dy}{dx}$ for $y = \sin\frac{1}{x}$

There are **other inverse trigonometric functions**. Here are their derivative formulas:

$$\frac{d}{dx}\sec^{-1}\boldsymbol{x} =$$

$$\frac{d}{dx}\cos^{-1}\boldsymbol{x} =$$

$$\frac{d}{dx}\cot^{-1}\boldsymbol{x} =$$

$$\frac{d}{dx}\csc^{-1}\boldsymbol{x} =$$