3.7 Implicit Differentiation

Implicit differentiation is a special case of the chain rule for derivatives. Rather than rewriting a function in the form $y = f(x)$, the both sides of the equation are derived with respect to the same variable

Example 1

Find $\frac{dy}{dx}$ if $y^2 = x$ using implicit differentiation.

What does this mean graphically?

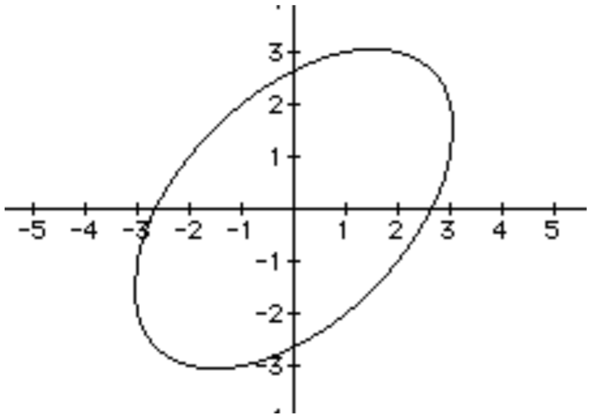
Consider the slope of the tangent lines at points (4, 2) and (4, –2), on the curve $y^2 = x$

Example 2

Find the slope of the circle $x^2 + y^2 = 25$ at the point (3, –4)

Example 3

Find the tangent and normal to the ellipse $x^2 - xy + y^2 = 7$ at the point (–1, 2)



Example 4

Show that the slope $\frac{dy}{dx}$ is defined at every point on the graph of $2y = x^2 + \sin y$

Example 5

Find $\frac{dy}{dx}$ if

a) $x \sin y = \cos(x + y)$

b) $6x^2 + 3xy + x^2y - 6y = 0$

Example 6

Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 8$

Try:

Find the slope of the tangent line of the cardioid:
$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at (0, ½)