3.3 The Factor Theorem

Factor Theorem:

If a polynomial $P(x)$ is divided by $(x - a)$ and the remainder is zero, then $(x - a)$ is a factor of $P(x)$.

In other words, if $P(a) = 0$, then $(x - a)$ is a factor of P(x).

Example:
Does $P(x) = x^3 + 2x^2 + 4x + 8$ have a factor of $(x + 2)$?

Rational Root Theorem (Possible Roots):

The possible roots of polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0$ are :

$\frac{\text{Factors of } a_0}{\text{Factors of } a_n}$

Example 1:

$2x^3 - 5x - 9$ has possible roots:

$\frac{\text{Factors of }}{\text{Factors of }} =$

Factoring Polynomials:
To factor a polynomial, we must use the rational root theorem and factor theorem to determine the roots.
Using the possible roots, we can try to find a root that satisfies the factor theorem; in other words, find a value of "a" such that P(a) = 0.

<u>Steps:</u> Factoring (or solving) an n>2 degree polynomial.

1. Find all possible roots
2. Find a root that satisfies P(a) =0; start testing with smaller values.
3. Synthetically divide the factor.
4. Repeat until completely factored; do not forget to include all roots.

*Note: Factors can always be repeated!

Example 2:
Factor completely:

a.) $x^3 \ -2x^2 \ -13 \ - \ 10$

b.) $2x^3 \ -7x^2 \ -7x \ + \ 12$

c.) $x^3 - 3x^2 + 3x - 1$

d.) $2x^3 - 5x^2 - 4x + 3$

e.) $x^4 - 5x^3 + 2x^2 + 20x - 24$

Example 3:
Solve by factoring:

a.) $x^4 - 3x^3 + x^2 + 3x - 2 = 0$

b.) $x^4 + 4x^3 + 2x^2 - 5x - 2 = 0$

Examples 4:

1. A box is constructed such that the length is three times the width and the height is 3 cm longer than the width, with a volume of 600 cm^3. What are the dimensions of the box?

2. The product of three consecutive odd integers is 105. What are the numbers?

P 133: 1-16