3.3 The Factor Theorem

Factor Theorem:

If a polynomial \( P(x) \) is divided by \( (x - a) \) and the remainder is zero, then \( (x - a) \) is a factor of \( P(x) \).

In other words, if \( P(a) = 0 \), then \( (x - a) \) is a factor of \( P(x) \).

Example:

Does \( P(x) = x^3 + 2x^2 + 4x + 8 \) have a factor of \( (x + 2) \)?

Rational Root Theorem (Possible Roots):

The possible roots of polynomial \( P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0 \) are:

- Factors of \( a_0 \)
- Factors of \( a_n \)

Example 1:

\( 2x^3 - 5x - 9 \) has possible roots:

Factors of \( -9 \) =
Factors of \( 2 \)

Factoring Polynomials:

To factor a polynomial, we must use the rational root theorem and factor theorem to determine the roots.

Using the possible roots, we can try to find a root that satisfies the factor theorem; in other words, find a value of "a" such that \( P(a) = 0 \).

Steps: Factoring (or solving) an n>2 degree polynomial.

1. Find all possible roots
2. Find a root that satisfies \( P(a) = 0 \); start testing with smaller values.
3. Synthetically divide the factor.
4. Repeat until completely factored; do not forget to include all roots.

*Note: Factors can always be repeated!
Example 2:
Factor completely:

a.) $x^3 - 2x^2 - 13 - 10$

b.) $2x^3 - 7x^2 - 7x + 12$
c.) $x^3 - 3x^2 + 3x - 1$

d.) $2x^3 - 5x^2 - 4x + 3$
e.) $x^4 - 5x^3 + 2x^2 + 20x - 24$

**Example 3:**
Solve by factoring:

a.) $x^4 - 3x^3 + x^2 + 3x - 2 = 0$
b.) \(x^4 + 4x^3 + 2x^2 - 5x - 2 = 0\)

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**Examples 4:**

1. A box is constructed such that the length is three times the width and the height is 3 cm longer than the width, with a volume of 600 cm\(^3\). What are the dimensions of the box?
2. The product of three consecutive odd integers is 105. What are the numbers?