3.1 Characteristics of Polynomial Functions

A **polynomial function** is the addition or subtractions of terms that contain variables and constants.

$a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x^1 + a_0$

Where $a_n \in \mathrm{R}$, $n \in Z^+$

Degree: the exponent to the highest power of x, which is n

Leading coefficient: the number in front of the highest power of x, a_n

Constant Term: the term without a variable, which is a_0. Also the value when $x = 0$

Example 1:

Polynomial	Degree	Leading Coefficient	Name
$f(x) = 3$			
$g(x) = \dfrac{2x-5}{2}$			
$h(x) = -2x^2 + 4x - 5$			
$j(x) = 0.3x^3 + \sqrt{3}x + 2$			
$k(x) = -\sqrt{5}x^3 + 4x - 5$			

Non-Polynomial	Reason
$f(x) = 3x^{-2}$	
$g(x) = 2\sqrt{x}$	
$h(x) = -2x^{0.5} - 5$	
$j(x) = \dfrac{2x^2 - 5x}{x}$	
$k(x) = \sqrt{-5}x^3 - 5$	

The **end behaviour** of a polynomial function is the y-value of the function as the x-value approaches +∞ and -∞.

The end behaviour is based on the leading term of the polynomial; specifically the degree and whether the leading coefficient is positive or negative.

Graphs of Polynomial Functions

1) Constant Function

Degree:

Number of intercepts:

End behaviour:

Domain:

Range:

2) Linear Function

Degree:

Number of intercepts:

End behaviour:

Domain:

Range:

3) Quadratic Function

Degree:

Number of intercepts:

End behaviour:

Domain:

Range:

4) Cubic Function

Degree:

Number of intercepts:

End behaviour:

Domain:

Range:

5) Quartic Function

6) Quintic Function

Degree:

Number of intercepts:

End behaviour:

Domain:

Range:

Degree:

Number of intercepts:

End behaviour:

Domain:

Range:

What patterns do you notice about these graphs?

What happens if $a_0 < 0$?

Maximum – the largest y-value(s) for the function

Relative Maximum – largest y-value of all the points around it
Absolute Maximum – largest y-value for all points in the function

Minimum – the smallest y-value

Relative Minimum – smallest y-value of all points around it
Absolute Minimum – smallest y-value for all points in the function

Zeros – value(s) of x when y = 0
where the graph hits the x axis
x-intercept
roots

Summary:

Polynomial functions can fit in three categories: constant, odd or even. They are continuous and are smooth (no edges).

Constant:

Odd:

Even:

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