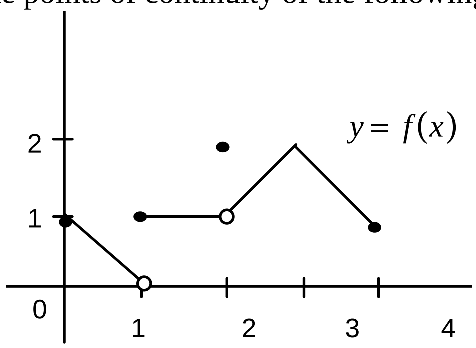
2.3 Continuity

Continuity at a Point

Definition: Continuity at a Point
A function $y = f(x)$ is continuous at an ***interior point c*** of its domain if $\lim_{x \to c} f(x) = f(c)$.
Endpoint: A function $y = f(x)$ is ***continuous at left endpoint a*** or is ***continuous at right endpoint b*** of its domain if $\lim_{x \to a^+} f(x) = f(a)$ or $\lim_{x \to b^-} f(x) = f(b)$, respectively.

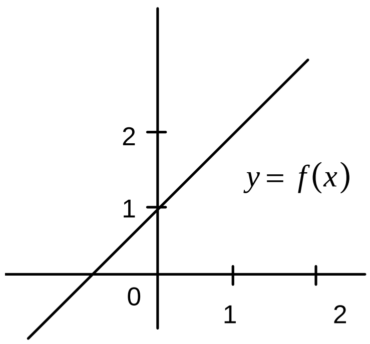
Example 1: Finding Points of Continuity and Discontinuity
Find the points of continuity of the following graph.



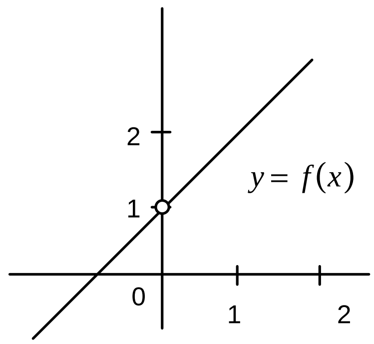
Removable vs Non-removable

f has a removable discontinuity if the function can be made continuous by redefining *f(c)* (such as a hole)

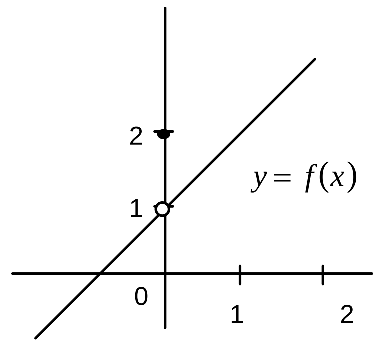
An Analysis of Some Important Functions and Types of Discontinuity



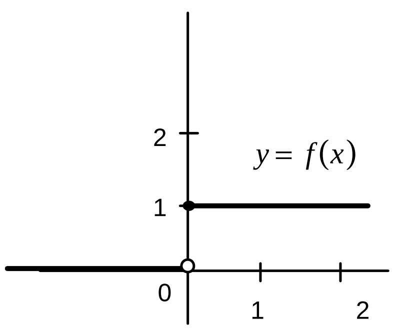
a) continuous at $x = 0$



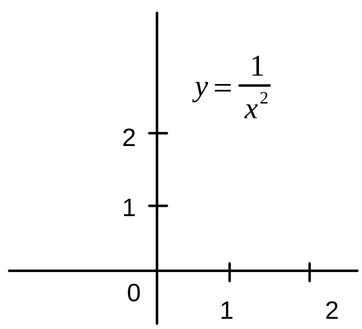
b) **removable discontinuity**



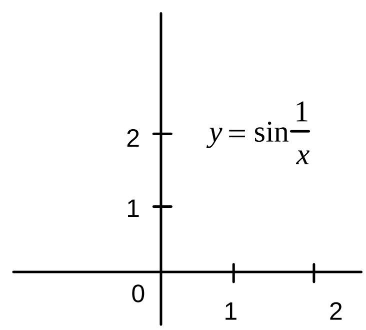
c) **removable discontinuity**



d) **jump discontinuity**



e) **infinite discontinuity**



f) **oscillating discontinuity**

Example 1: Let $\boldsymbol{f(x)} = \frac{\boldsymbol{x^3 - 7x - 6}}{\boldsymbol{x^2 - 9}}$

a.) Is the function continuous? If not, what kind of discontinuity is there?

b.) Can you create an *extended function* $\boldsymbol{g(x)}$ through a piecewise function that fills the hole of discontinuity?

Continuous Function:

A function is **continuous on an interval** if and only if it is continuous at every point of the interval.

A **continuous function** is one that is continuous at every point of its domain. A continuous function need not be continuous on every interval. For example, $\boldsymbol{y = f(x)}$ is not continuous on $[-1, 1]$ but is a continuous function.

Non-technical terms: a function is continuous if you can draw the function without lifting your pencil.

Intermediate Value Theorem for Continuous Functions

A function $y = f(x)$ that is continuous on an closed interval $[a, b]$, takes on every value between $f(a)$ and $f(b)$.

Example 2: Using the Intermediate Value Theorem
Is any real number exactly 1 less than its cube?

Example 3:

Let $\boldsymbol{f(x) = \frac{x^3+x}{x-1}}$. Verify that the IVT applies to the interval [2.5, 4] and find the value of *c* guaranteed by the theorem if $\boldsymbol{f(c) = 6}$