10.3 Composite Functions

The composition of $f(x)$ and $g(x)$ is defined as $f(g(x))$ and is formed when $g(x)$ is substituted into $f(x)$ (or $(f o\, g)\,(x)$)

Example 1: Evaluate the following

If $f(x) = x^2 + x$ and $g(x) = 5x$ find

a.) $f(g(3))$

b.) $g(f(-2))$

c.) $f(g(-5))$

d.) $f(g(f(1)))$

e.) $f(g(x))$

f.) $g(f(x))$

Domain of Composite functions

The domain of $f(g(x))$ only exists for those x in the domain of g for which $g(x)$ is in the domain of f.

Example 2: Functions with restrictions

If $f(x) = x^2$ and $g(x) = \sqrt{x}$, find

a.) $f\left(g(x)\right)$

b.) $g(f(x))$

Transformations as a Form of Composite Functions

The transformations from chapter 1 are forms of composite functions: we can replace x and y with different functions and our graph changes relative to the change in our functions.

ie: A horizontal compression by 2 on a function $y = f(x)$ would be written as $y = f(2x)$

Example 3:

Given the following functions, $y = f(x)$, rewrite the following transformations in the form $y = af\left(b(x-h)\right) + k$ then rewrite the equation using compositions.

a.) $y = \sqrt{x}$ is expanded vertically by 2, expanded horizontally by 3 and translated 3 left

b.) $y = \frac{1}{x}$ is vertically compressed to $\frac{1}{2}$, reflected across the x-axis, moved right 5 and down 3.

c.) $y = x^3 - 2x^2$ is reflected across the x-axis and translated 3 units right.

d.) $y = \frac{1}{x^3}$ is reflected across the y-axis, compressed horizontally to a factor of $\frac{1}{2}$ and expanded vertically by a factor of 5.

e.) $y = x^2 + x$ is compressed horizontally to a factor of $\frac{1}{3}$ and shifted 10 units left.

f.) $y = x^2 + x$ is shifted 10 units left THEN is compressed horizontally to a factor of $\frac{1}{3}$