1.4 Inverse of a Relation

The inverse of a relation is found by interchanging all x-coordinates with y-coordinates. The graph is a reflection across the line $y = x$

The mapping of an inverse is given by:

$$(x, y) \rightarrow (y, x)$$

Notation:

$f^{-1}(x)$ is the inverse function of $f(x)$

Graphing Inverse Functions

To graph the inverse of the function, find coordinates and interchange the x and y values.

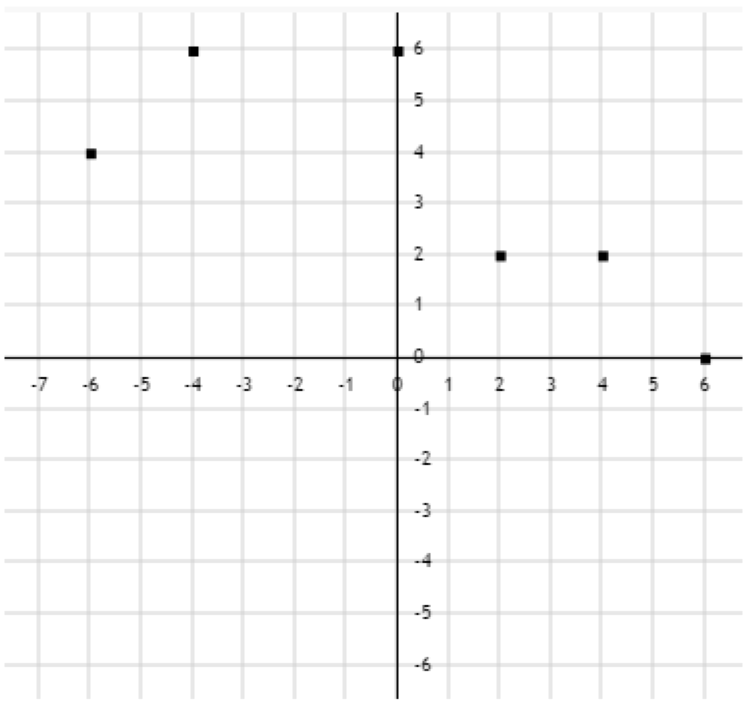
The graph is also a reflection across the $y = x$ line.

Example 1:

a.) Sketch the graph of the inverse relation.

b.) State the domain and range of the inverse relation.

c.) Determine whether the relation and its inverse are functions.



Vertical Line Test:

The graph is a function if at every x coordinate; the vertical line that passes through the x coordinate only passes through the graph at most one time.

Horizontal Line Test:

If the graph satisfies the horizontal line test, the inverse will be a function (because it will satisfy the vertical line test).

One to one function:

If the graph satisfies both the vertical and horizontal line test it is called a one-to-one function; where every element in the range corresponds to exactly one element in the domain

Restriction of the Domain:

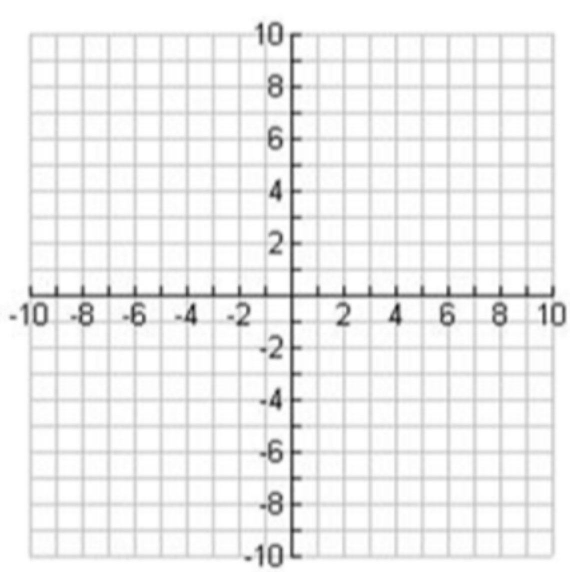
In some cases, by restricting the domain of the graph, we can force the inverse to be a function.

Example 2:

Consider and graph $y = (x - 3)^2$

a.) Graph the inverse.

b.) What restrictions can we apply to the original graph that makes the inverse a function?



To find the inverse analytically:

1. Interchange all x with y

2. Solve for y

3. $y = f^{-1}(x)$

4. Domain/Range of $f(x)$ becomes Range/Domain of $f^{-1}(x)$

Example:

Determine the inverse of the following:

a.) $f(x) = 3x + 2$

b.) $f(x) = \sqrt{x+3} - 2$

c.) $f(x) = \frac{3x}{x+2}$